U.S. Department of
Transportation

**Federal Railroad
Administration**

Mixed Freight and Higher-Speed Passenger Trains: Framework for Superelevation Design

Office of Research,
Development
and Technology
Washington, DC 20590

DOT/FRA/ORD-19/42

Final Report
October 2019

REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 0704-0188*

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE October 2019	3. REPORT TYPE AND DATES COVERED Technical Report, September 2015 – May 2016

4. TITLE AND SUBTITLE Mixed Freight and Higher-Speed Passenger Trains: Framework for Superelevation Design	5. FUNDING NUMBERS DTFR53-15-C-00023
6. AUTHOR(S) Tyler Dick, Conrad Ruppert	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) University of Illinois at Urbana-Champaign 205 North Mathews Avenue, MC-250 1245A Newmark Civil Engineering Laboratory Urbana, IL 61801	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) U.S. Department of Transportation Federal Railroad Administration Office of Railroad Policy and Development Office of Research, Development and Technology Washington, DC 20590	10. SPONSORING/MONITORING AGENCY REPORT NUMBER DOT/FRA/ORD-19/42
11. SUPPLEMENTARY NOTES COR: Cameron Stuart	
12a. DISTRIBUTION/AVAILABILITY STATEMENT This document is available to the public through the FRA eLibrary.	12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

As the demand for higher-speed passenger trains increases, the disparity between freight and passenger operations also increases. On mixed-use rail corridors, the result is a bi-modal distribution of train speeds that complicates design of curve superelevation. To aid in making informed curve superelevation decisions on mixed-use rail corridors, this report synthesizes industry knowledge and tools used to evaluate curve superelevation. This report develops the superelevation bandwidth concept into a framework to optimize superelevation on mixed-use corridors. The framework is presented as both a graphical approach and a mathematical model that can be applied to select superelevation design parameters for multiple curves on a rail corridor. Five case studies illustrating typical design situations encountered in the development of mixed-use corridors are highlighted. Each case study demonstrates how the selection of actual superelevation results in trade-offs between the operation of slower trains with overbalance and imposing curve speed restrictions on faster trains.

14. SUBJECT TERMS Superelevation, high-speed rail, freight operations, passenger operations	15. NUMBER OF PAGES 65
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT

Contents

Illustrations

Tables

Executive Summary

As the demand for higher-speed passenger trains increases, the disparity between freight and passenger operations also increases. Higher-speed passenger trains use improved suspensions, low center of gravity, radially steering trucks, and tilting technology, allowing them to operate at higher speeds through curves. The co-operation of long, heavy-axle-load freight trains can result in much lower speeds on the same curve. The result is a bi-modal distribution of train speeds on mixed-use rail corridors that complicates curve superelevation design. The goal of this research project was to synthesize industry knowledge and tools to aid rail operators in making informed curve superelevation decisions on mixed-use rail corridors. This research was conducted by University of Illinois at Urbana Champaign (UIUC) between September 2015 and May 2016, and was funded by the Federal Railroad Administration (FRA).

The disparity in train speeds on mixed-use corridors may be due to the different business objectives of certain types of freight and passenger service, the curving capability of different types of rail vehicles, and local site conditions that may cause certain trains to negotiate nearby curves at less than normal timetable operating speeds. Since a given curve has one unique equilibrium speed, a range of curving speeds results in both underbalance and overbalance operating conditions which impact track safety, maintenance, ride comfort and network capacity. Under these conditions, quasi-static vertical wheel loads can be increased by over 20 percent, increasing maintenance. These conditions can be exacerbated on grades where in-train forces can increase or decrease required superelevation. In particular, the overbalance condition should be avoided, as it promotes rolling contact fatigue.

The superelevation design criteria specified by each railway vary in their maximum allowable actual superelevation and maximum allowable cant deficiency. The maximum allowable curving speed for a particular type of train and curve design can vary among railways.

On a given curve, a design superelevation “bandwidth” can be defined and compared to the frequency distribution of train speeds operating on the route. Trains falling below the lower bound set by the equilibrium speed will operate in an undesirable overbalance condition. Trains falling above the upper-bound speed set by maximum cant deficiency will be subject to civil curve speed restrictions. A combination of actual superelevation and allowable cant deficiency that satisfies railway design criteria while maximizing the number of trains falling within the superelevation bandwidth will provide the best solution for a mixed-use corridor. Using the bandwidth approach, trains can be weighted by traffic in terms of freight tonnage or number of passenger trains to better reflect the maintenance and revenue implications of operating outside the design superelevation bandwidth.

This report develops the superelevation bandwidth concept into a framework to optimize superelevation on mixed-use corridors. The framework is presented as both a graphical approach and a mathematical model that can be applied to select superelevation deign parameters for multiple curves on a rail corridor.

This report explores five case studies to illustrate typical design situations encountered in the development of mixed-use corridors. Each case study demonstrates how the selection of actual superelevation results in trade-offs between the operation of slower and faster trains.

1. Introduction

The purpose of this research is to provide railway corridor planners with framework for the optimal design of superelevation for curves serving both freight and passenger train operations. UIUC researchers synthesized information on how the selection of superelevation affects running time performance, safety, ride comfort, and maintenance, and established a framework to evaluate the trade-offs between these factors. The tools and methodologies presented in this document can be used by practitioners to make more informed decisions regarding superelevation design of railway curves.

1.1 Background

The basic physics of establishing balanced superelevation on curves for a single type of rail vehicle operating at a constant speed is well-documented in railway texts and design literature. However, the situation becomes more complex when a curve is subject to trains operating over a wide range of speeds. When operating at speeds above or below the balancing speed for the superelevation on the curve, rail vehicles may impart additional vertical and lateral loads into the track infrastructure—with safety, maintenance and ride comfort implications. Although this is not a new phenomenon, the range of maximum freight and passenger train speeds on a railway curve in North America has historically been relatively narrow. Many conventional passenger trains only operate 10 to 15 mph faster than priority freight trains on curves due to limits on cant deficiency, vehicle suspensions, and other railroad design and operating parameters.

Recent trends toward higher-speed passenger rail service have increased the disparity in current and planned maximum train operating speeds on many rail corridors. The higher-speed passenger trains often use improved suspensions, lowered center of gravity and tilting technology to operate at higher cant deficiencies and faster speeds on curves. An extreme example is Amtrak's Northeast Corridor (NEC) where the difference in operating speed through curves between passenger and freight trains can be 80 mph or more. At the same time, efforts to seek even greater efficiencies have led to the operation of heavy-axle-load freight trains of increasing length and total weight. Especially on grades, these long, heavy freight trains often operate below maximum allowable freight train speeds. On shared or mixed-use corridors, the result of these operational trends can be a bi-modal distribution of train speeds on curves. Selection of a single optimal value for curve superelevation under these conditions is not straightforward.

1.2 Objectives

The objective of this research was to develop a manual that will provide railway corridor planners with a framework to determine the optimal design of superelevation on curves serving both freight and passenger train operations. The research aims to develop a comprehensive understanding of how selection of superelevation impacts running time performance, safety, ride comfort and maintenance, and establish a framework to evaluate the trade-offs between each of these factors. The results of this research identifies the tools and methodologies that practitioners can use to make more informed decisions regarding design superelevation on curves.

1.3 Overall Approach

The approach to this research included multiple elements. First, the project team reviewed existing texts to develop a comprehensive and consistent description of the physics of superelevation and its role in safe and comfortable negotiation of curves. They also reviewed the research literature to better understand the implications of operating trains at speed well above and below the balanced speed. Second, the project team compiled standard superelevation, spiral and unbalance design approaches and evaluation tools used by major passenger and freight railroads, both in North America and internationally, in curve design and corridor planning. Third, the project team developed a framework that relates superelevation selection to various engineering and performance considerations. Finally, the project team developed case studies to illustrate application of the framework to typical design situations encountered in the development of shared corridors.

1.4 Scope

The discussion of railway curve superelevation in this document is focused on mainline standard-gage railways in North America with a emphasis on mixed-use corridors with both freight and passenger train operations. The problem of superelevation design for multiple train speeds is approached from both the perspective of a private freight railroad owner and a public passenger/agency owner. The mixed-use corridors may range from those dominated by freight traffic to those dominated by passenger operations. International experience on mixed-use corridors within comparable operating environments is also considered in the review of literature and current design practice.

This discussion does not consider dedicated high-speed rail lines as these systems typically have a very narrow range of operating speeds on curves.

This discussion is also limited to rail lines under jurisdiction of the Federal Railroad Administration, including commuter rail operations. This document does not address curve superelevation of heavy rail, light rail or other types of rail transit operations. Like high-speed rail, transit systems typically operate at more consistent speeds on curves with a more uniform fleet of rolling stock. Transit systems may also operate on non-standard gage track, changing the fundamental physics and equations governing superelevation design.

The materials presented in this document are intended to aid practitioners in making more informed decisions regarding design of superelevation on railway curves. This document should not be interpreted as a design standard or required approach. In all cases, individual railway and operator standards, design criteria, and other requirements will supersede the information presented in this document. The design criteria summarized in Section 3 are presented for informational purposes and are only considered accurate, complete, and current per the date of report publishing. Practitioners should obtain the latest track standards from the relevant railway or operator to verify superelevation criteria prior to design or maintenance activities.

1.5 Organization of the Report

Section 2 of this document reviews the basic physics of superelevation design and synthesizes knowledge on factors influencing the selection of superelevation to aid in making informed curve superelevation decisions on mixed-use rail corridors. The factors considered include running time performance, safety, vehicle-track dynamics, ride comfort, and track maintenance.

This discussion is followed a by a summary of current railway superelevation design criteria in Section 3. The summarized design criteria include those used by major passenger and freight railroads, both in North America and internationally. In Section 4, the compiled information is used to develop a graphical framework and mathematical model for optimizing superelevation on curves subject to disparate train speeds. Section 5 presents five case studies to illustrate the application of the framework to typical design situations encountered in the development of shared corridors.

2. Physics of Superelevated Railway Curves

This section reviews the basic physics of superelevation design for railway curves as background for subsequent discussion. This is followed by a description of how traversing superelevated curves at different speeds changes the distribution of lateral and vertical forces acting on each rail. The influence of in-train forces from grades on superelevation is also introduced. Information on the maintenance and safety considerations of superelevation design is also synthesized to reinforce the importance of proper superelevation design.

2.1 Basic Quasi-Static Curving Analysis

The basic physics of superelevation is well understood and presented in numerous textbooks and other references, with Elkins and Gostling (1977) as one example. Moody (2014) presented an excellent summary of the basic physics and equations governing superelevation and unbalance on railway curves.

To negotiate a circular curve at constant speed, a rail vehicle must be subject to a centripetal force acting inwards toward the center of the curve. The magnitude of this force is a function of the sharpness (radius or degree) of the curve, the speed of the rail vehicle, and the mass (weight) of the rail vehicle. The centripetal (or curving) force is created by a combination of the rail vehicle wheels reacting against the rails (lateral forces) and superelevation (or banking) on the curve. Superelevation raises the outside rail on the curve by rotating the plane of the track structure about the inside rail. When superelevated, the force on each rail normal to the plane of the track has a vertical component that acts against the force of gravity and a horizontal component that acts to push a rail vehicle toward the center of the curve, helping to create the necessary centripetal curving force.

On a railway curve, superelevation is measured as the difference in elevation between the low and high rail on the curve. For a given radius (degree) curve, the centripetal force required by a rail vehicle operating at a given speed will be exactly equal to the horizontal components of the normal rail reactions when the track is inclined at a certain superelevation (Figure 1). Under this combination of speed and superelevation, the rail vehicle is in equilibrium. The corresponding amount of superelevation is termed the *equilibrium superelevation* (or *balanced superelevation*) for that combination of speed and degree of curve.

As depicted in Figure 1, the angle (α) of the track plane created by the superelevation is equal to the angle between the normal rail force reactions and the gravitational force. For the equilibrium conditions depicted in Figure 1, two similar triangles are formed. By these similar triangles, for small values of superelevation, the ratio of superelevation (E) to the track gage (G) is equal to the ratio of the centripetal force resultant to the gravitational force. By this equivalency, the equilibrium superelevation (E) can be calculated by Equation 1 where G is the track gage distance between the rails, V is the speed of the rail vehicle, R is the radius of the curve, and g is the acceleration due to gravity (AREMA, 2015). The mass of the rail vehicle cancels out of the equations.

$$E = \frac{V^2}{gR} = \text{Equilibrium Superelevation} \qquad (1)$$

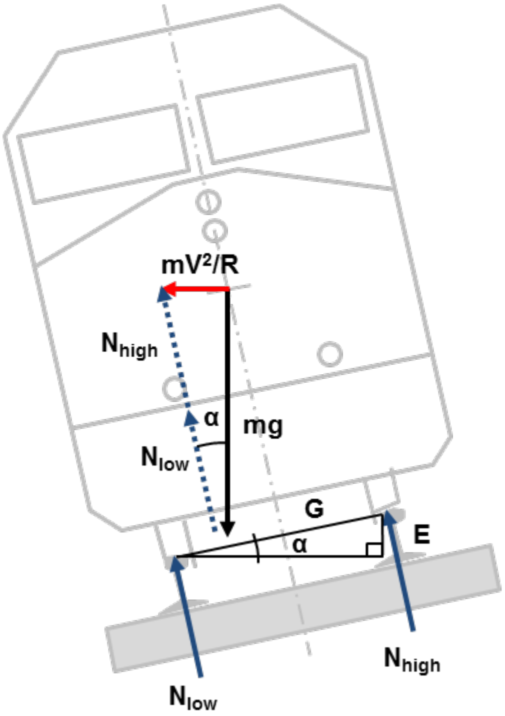


Figure 1: Forces on rail vehicle at equilibrium condition produce resultant centripetal force

By substituting the value of g, the arc definition of degree of curve for R, and making adjustments for units, Equation 1 can be transformed into Equation 2, relating the equilibrium superelevation (E_e) in inches to the degree of curve (D) and train speed (V) on standard-gage track where the track gauge (G) is equal to 56.5 inches) (*1*).

$$\mathrm{E_e} = 0.0007\mathrm{D}V^2 \qquad (2)$$

It is common practice to superelevate railway curves to some value less than the equilibrium superelevation. Equation 3 shows the simple equation for equilibrium superelevation, made up of the sum of E_a, the actual superelevation (cant), and E_u, the unbalanced superelevation (cant deficiency).

$$\mathrm{E_e} = \mathrm{E_a} + \mathrm{E_u} \qquad (3)$$

E_a and E_u are further defined as follows:

- *Actual Superelevation (or Cant)* (E_a) is the actual difference in elevation between the high and low rails on a curved segment of track expressed in inches. This is the amount of superelevation installed in the track.
- *Unbalanced Superelevation (or Cant Deficiency)* (E_u) is the difference between the actual superelevation and the superelevation required to create equilibrium conditions for the considered combination of speed and degree of curve. Cant deficiency exists when a rail vehicle travels through a curve at a speed greater than the equilibrium speed of that curve (given the actual superelevation and degree of the curve). Expressed in inches, cant deficiency is also referred to as “underbalance.”

Rearranging the terms of Equation 2 and substituting E_e from Equation 3, the maximum operating speed through a curve of constant degree of curvature in miles per hour is shown in Equation 4 below.

$$V_{max} = \sqrt{\frac{E_a+E_u}{0.0007D}} \qquad (4)$$

For a rail line with uniform traffic, actual superelevation on each curve can be designed for the maximum allowable track speed using Equation 5.

$$E_a = (0.0007DV_{max}^2) - E_u \qquad (5)$$

As noted previously and for reasons that will be discussed later in this document, the design speed is usually set such that trains operate with 1 or more inches of cant deficiency. Thus, the actual installed superelevation is usually 1 or more inches less than the superelevation required to obtain equilibrium conditions for the design speed.

2.2 Distribution of Lateral and Vertical Wheel Forces

The distribution of normal and lateral wheel loads changes when a train operates above or below the equilibrium speed on a superelevated curve (Figure 2). In this figure, the red arrow acting on the vehicle mass center is the net resultant centripetal force that is accelerating the vehicle around the curve. The black arrow in each scenario is the force due to gravity acting on the vehicle mass center. The dashed black lines illustrate track frame vertical and lateral components of the gravitational force. The blue arrows represent the vertical and lateral forces acting on the rail vehicle at the wheel/rail interface.

At equilibrium speed, the high and low rail vertical forces are equal in magnitude and there is no lateral force at the wheel/rail interface. In an *overbalance condition*, where cant excess exists, the low rail vertical force is greater in magnitude than high rail vertical force. In this condition, there is also a lateral force acting outward on the rail vehicle from the low rail. In an *underbalance condition*, where cant deficiency exists, the high rail vertical force is greater in magnitude than that of the low rail. In addition, there is a lateral force that acts inward on the rail vehicle from the high rail. The corresponding sets of force vector additions demonstrate how the different forces combine to produce a resultant force that guides the rail vehicle in a circular path around the curve at different speeds.

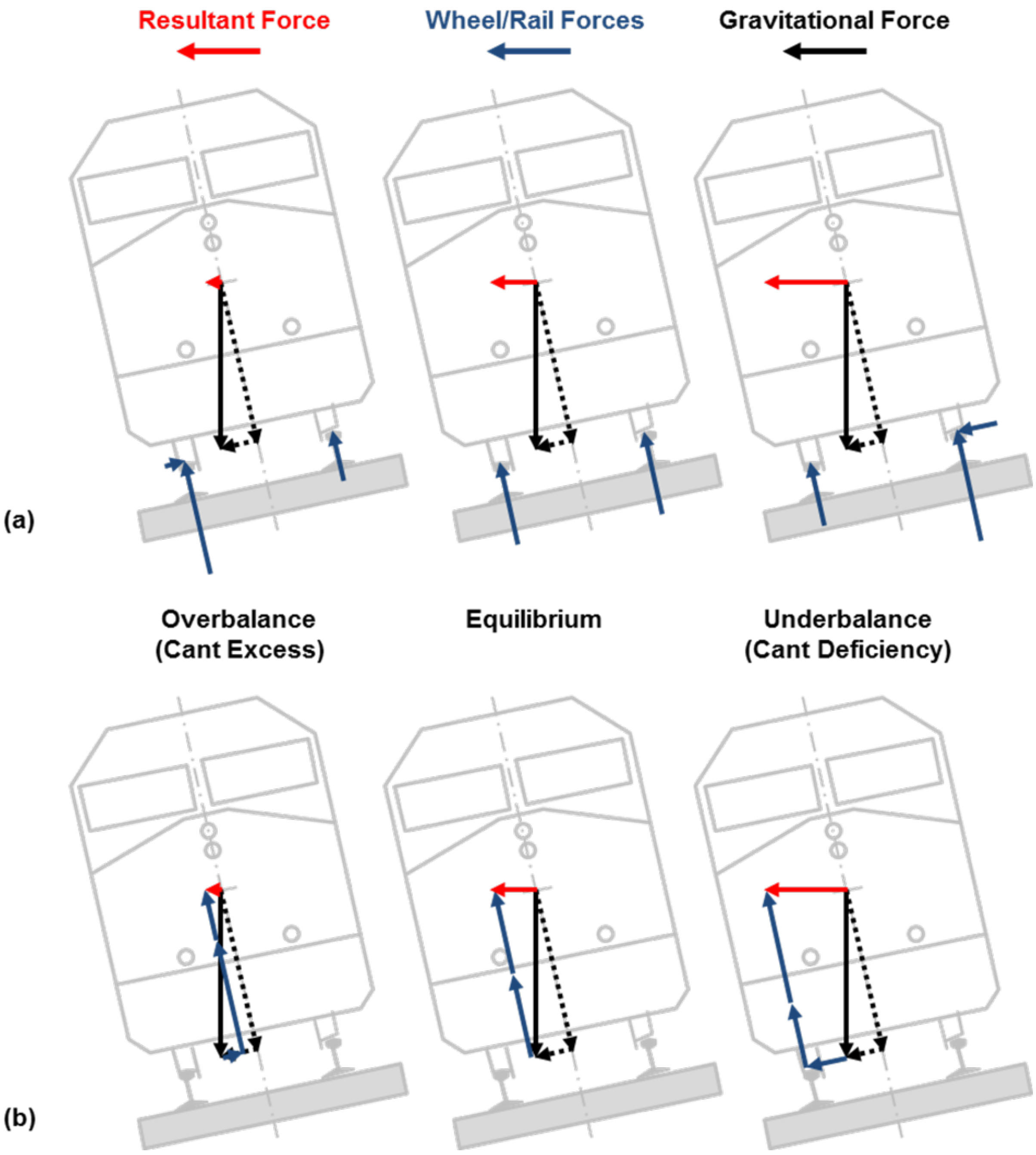


Figure 2: Illustration of (a) forces acting on a rail vehicle while curving at different speeds and (b) vector force addition to produce required centripetal resultant for circular motion

Summing forces perpendicular and parallel to the track in Figure 2 and taking moments about the rail vehicle center of gravity can yield Equations 6-8 for the vertical (normal) wheel load on each rail and the lateral wheel/rail force. In Equations 6-8, h is the height of the rail vehicle center of gravity above the top of rail. For typical railway equipment, h is assumed to be 7 feet.

$$N_{\text{low}} = \frac{mh}{}\left(\frac{gE}{} - \frac{V^2}{R}\right) + \frac{m}{2}\left(g + \frac{EV^2}{R}\right) = \text{Low Rail Normal Force} \qquad (6)$$

$$N_{\text{high}} = \frac{mh}{}\left(\frac{V^2}{R} - \frac{gE}{}\right) + \frac{m}{2}\left(g + \frac{EV^2}{R}\right) = \text{High Rail Normal Force} \qquad (7)$$

$$L = \frac{mgE}{} - \frac{mV^2}{R} = \text{Rail Lateral Force (positive outwards)} \qquad (8)$$

To illustrate the magnitude of the increased or decreased wheel loads under typical overbalance and underbalance conditions, Equations 6-8 were solved for a 3-degree curve over a range of train speeds and corresponding values of cant deficiency (negative cant deficiency is cant excess). Wheel loads were calculated for a loaded 286,000-lb car with a static wheel load of 35.75 kips.

The results of the calculation (Figure 3) indicated that both the vertical and lateral wheel loads increase linearly as cant deficiency is increased. This is equivalent to a parabolic relationship to train speed. At higher-speeds, small increases in train speed result in a greater change in wheel loads than at lower speeds. At the balancing speed of 43.6 mph, the forces on the high and low rail are equivalent to the static wheel load of 35.75 kips and the lateral force is negligible. When the train speed increases to a cant deficiency of 4 inches, the wheel load on the high rail increases by over 20 percent. Similarly, at a cant excess of 4 inches, the wheel load on the low rail increases by over 20 percent.

2.3 Consideration of In-Train Forces on Ascending and Descending Grades

One shortcoming of this classic approach is that it only looks at quasi-static forces acting on a single rail vehicle. If the rail vehicle is considered as part of a train, in-train buff and draft coupler forces become a factor in superelevation design.

When negotiating a curve, the couplers at either end of a railcar move to an angle relative to the longitudinal centerline of the railcar. When in-train buff and draft forces act in line with this angled coupler position, they create both a longitudinal force along the rail centerline and a lateral force. Typically, the coupler angle is very small and the lateral force is negligible. However, for heavy-haul freight trains negotiating curves on steep grades, the lateral component of the in-train forces can become large enough to aid or work against the desired curving motion (Tournay et al., 2014a-c). When this additional force is considered in the classic quasi-static condition, the amount of superelevation required to obtain equilibrium for a certain speed on a particular curve changes (Igwemezie, 2006).

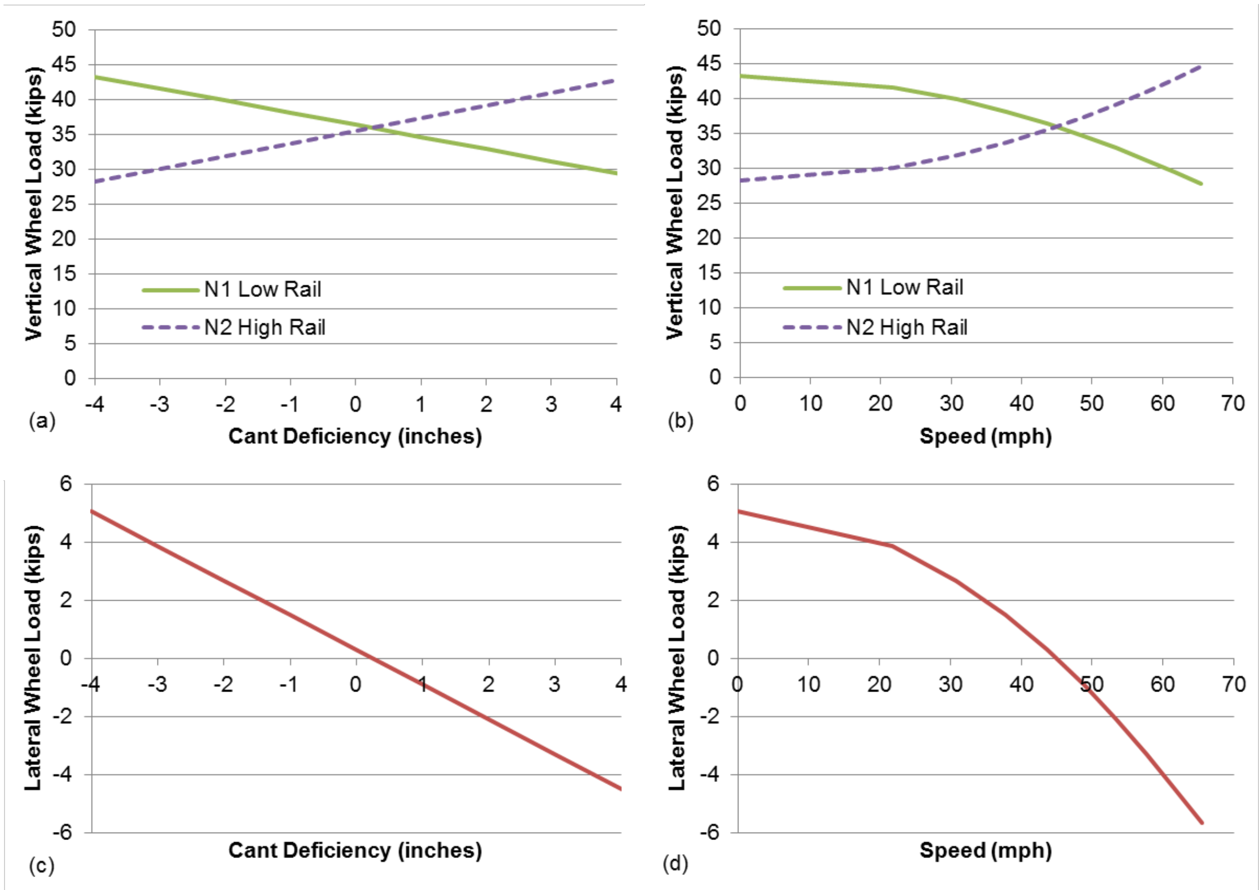


Figure 3: Variation in vertical and lateral wheel loads over a range of cant deficiency (a and c) and speed (b and d) for a 286,000-lb railcar on a 3-degree curve

2.3.1 Draft Forces on Ascending Grade

On an upgrade, assuming all motive power is at the front of the train, draft (or tension) coupler forces act at an angle to the railcar centerline in both directions (Figure 4a). The lateral component of these draft forces acts to pull the railcar inwards toward the center of the curve. As shown by the vector addition in Figure 4a, the draft forces help to achieve the necessary centripetal curving force and decrease the amount of force required from superelevation. With less force required, less superelevation is necessary to balance the train speed on the curve than in the quasi-static case that neglects in-train forces.

At slow speeds, draft forces are large and superelevation is set according to Equation 5; the combined coupler and superelevation forces may overcompensate for the speed of the train on the curve. To balance the system, additional outward lateral forces are required from reactions through the wheel/rail interface, potentially increasing wear on the low rail.

To demonstrate the magnitude of these lateral forces on upgrade freight operations, net horizontal forces have been calculated for the railcar at position 50 in a loaded, 100-railcar freight

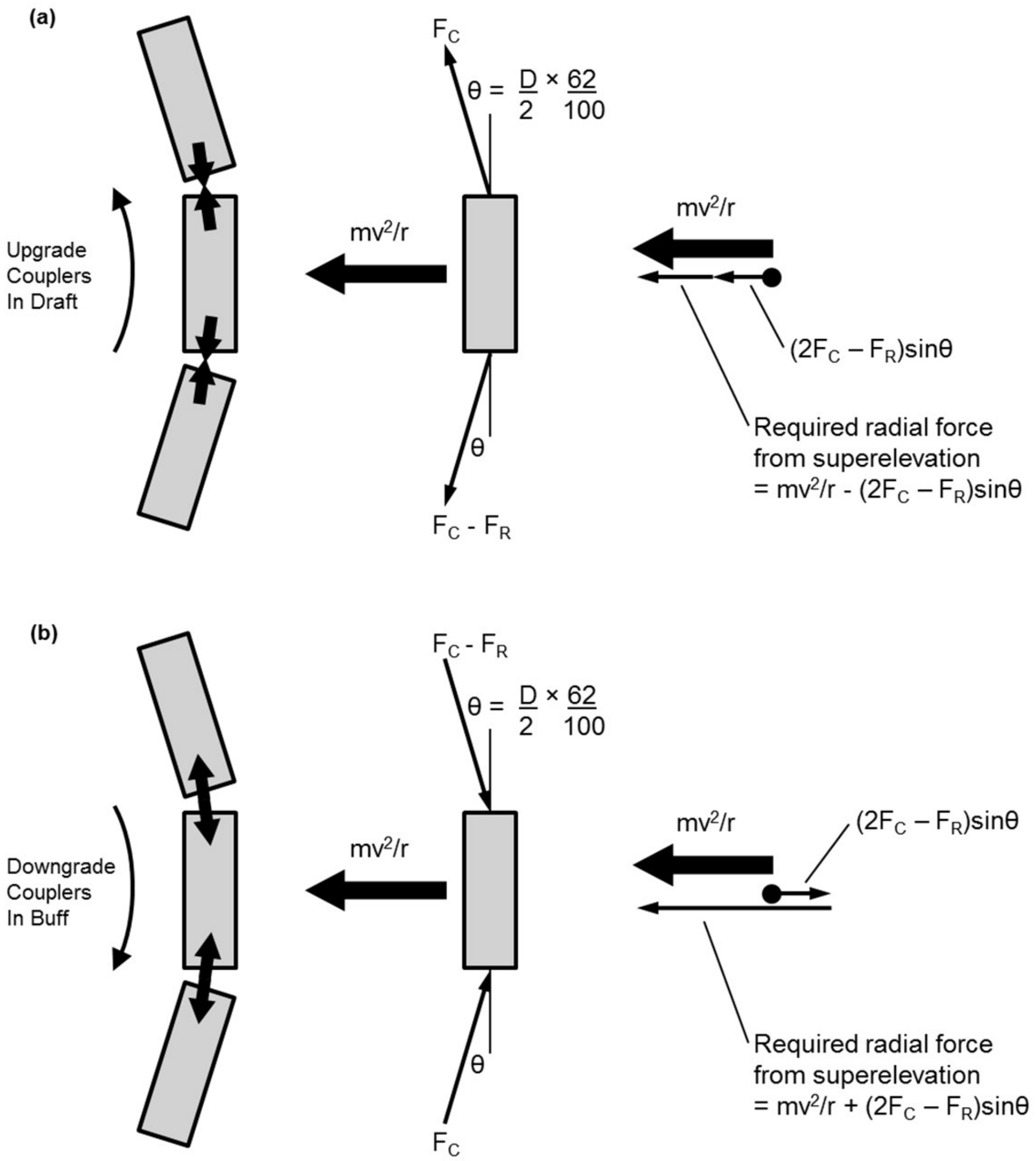


Figure 4: Influence of in-train forces on required superelevation for (a) upgrade and (b) downgrade conditions at the same train speed

train (286,000 lbs per railcar) operating over a range of speeds on a 1-percent upgrade and curves of different degree (Figure 5). At slower speeds, the net force is negative and there will be a lateral force reaction against the low rail. There is no need for superelevation, as the lateral coupler forces on the 50th railcar are larger than the centripetal forces required for circular motion around the curve. At higher speeds, the net force is positive. Although superelevation is required, the need for superelevation is reduced, as the lateral coupler forces act in the direction of the centripetal curving forces. Where the superelevation is not large enough to provide the required net force, there will be a lateral force reaction against the high rail.

Since coupler forces vary over the length of a train, each railcar within a train can have its own unique superelevation requirements. Net horizontal forces were calculated over the length of a loaded 100-railcar freight train travelling at a constant 15 mph (22 feet per second) on a 1-percent upgrade and several degree curves (Figure 6). Since all locomotives were assumed to be at the front of the train, the first railcar experiences the largest coupler forces and largest negative net horizontal forces. Forces then decrease (become less negative) over the length of the train. The absolute magnitude of the negative horizontal forces at the beginning of the train is far larger than the positive horizontal forces at the end of the train.

2.3.2 Buff Forces on Descending Grade

On a downgrade, under similar assumptions, draft (or compression) coupler forces act at an angle to the railcar centerline (Figure 4b). The lateral component of these buff forces acts to push the railcar outwards, away from the center of the curve. As shown by the vector addition in Figure 4b, the buff forces act against the necessary centripetal curving force and increase the amount of force required from superelevation. Thus more superelevation is required to balance the train speed on the curve than the original quasi-static case where in-train forces are neglected.

The need for additional superelevation is important in this case because trains descending grades are more likely to be travelling closer to the track design speed. At these speeds, the actual superelevation may not be sufficient to adequately balance the combined coupler and centripetal curving forces. To balance the system, additional inward lateral forces are required, potentially increasing rail wear on the high rail. In extreme cases, the lateral forces may cause the high rail to roll over, leading to a derailment.

To demonstrate the magnitude of these lateral forces on downgrade freight operations, net horizontal forces have been calculated for the railcar at position 50 in a loaded 100-railcar freight train operating over a range of speeds on a 1-percent upgrade and curves of different degree (Figure 7). In the downgrade case, the net force on the 50th railcar is always positive. The need for superelevation is increased as the lateral coupler forces act in the direction opposite the centripetal curving forces. As described above, where the superelevation is not large enough to provide the required net force, there will be a lateral force reaction against the high rail.

To demonstrate variation with train length, net horizontal forces were calculated over the length of a loaded 100-railcar freight train travelling at a constant 15 mph (22 feet per second) on a 1-percent downgrade and several degree curves (Figure 8). Since all locomotives were assumed to be at the front of the train, the first railcar experiences the largest coupler forces and largest net horizontal forces. Forces then decrease over the length of the train. The absolute magnitude of the negative horizontal forces at the beginning of the train is far larger than the positive horizontal forces at the end of the train.

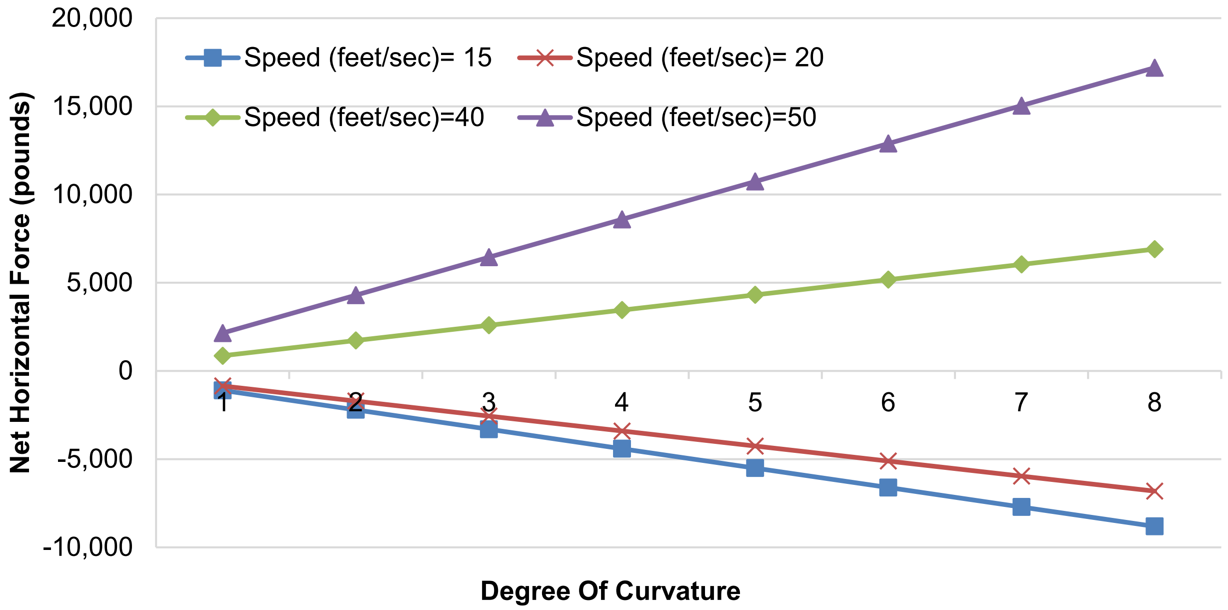


Figure 5: Variation in net horizontal forces on 50th railcar in a 100-car train on a 1-percent upgrade for various combinations of degree of curvature and train speed

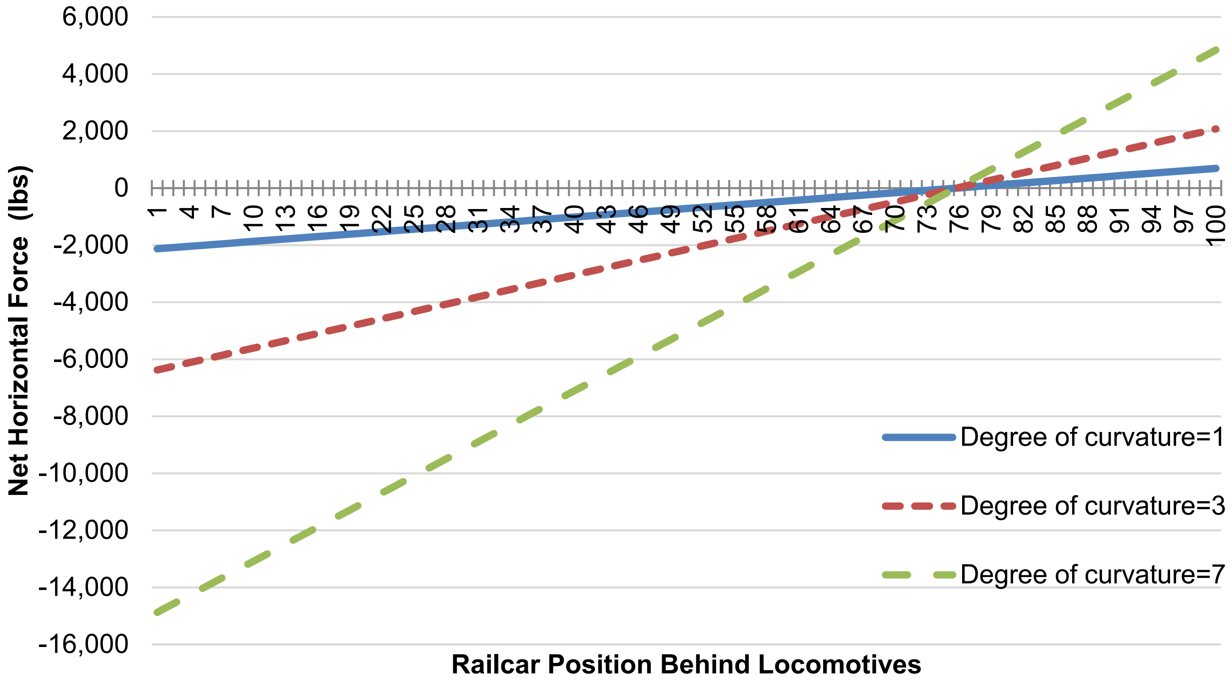


Figure 6: Variation in net horizontal forces in a 100-car freight train on a 1-percent upgrade for three different degrees of curvature

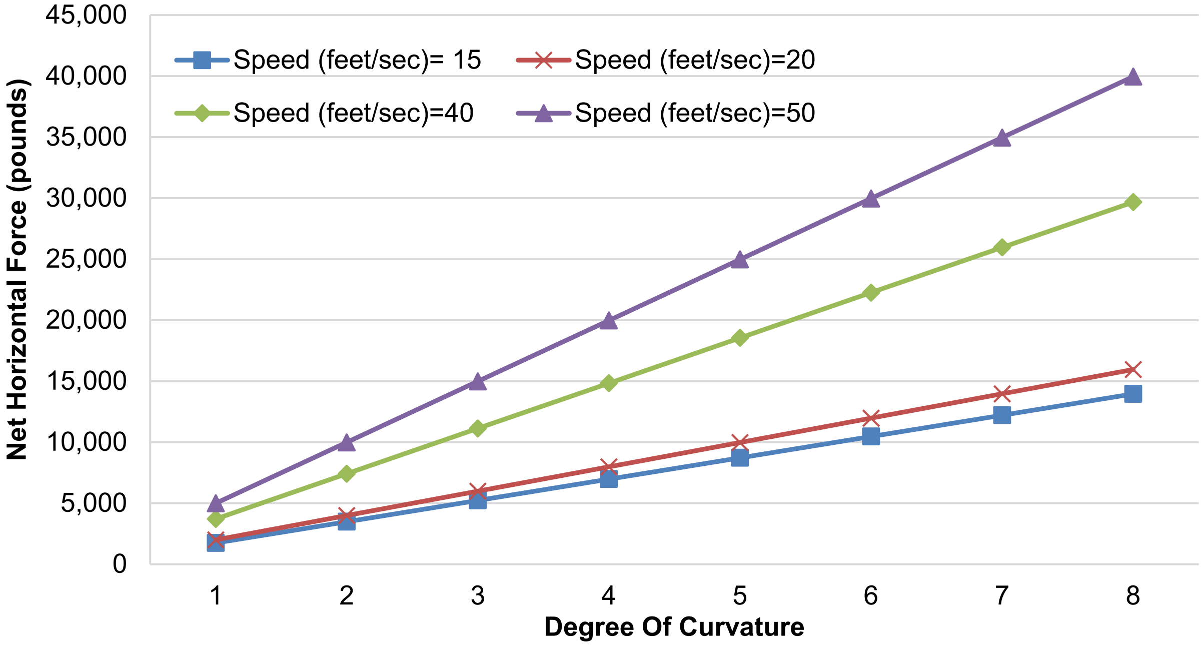


Figure 7: Variation in net horizontal forces on 50th railcar in a 100-car train on a 1-percent downgrade for various combinations of degree of curvature and train speed

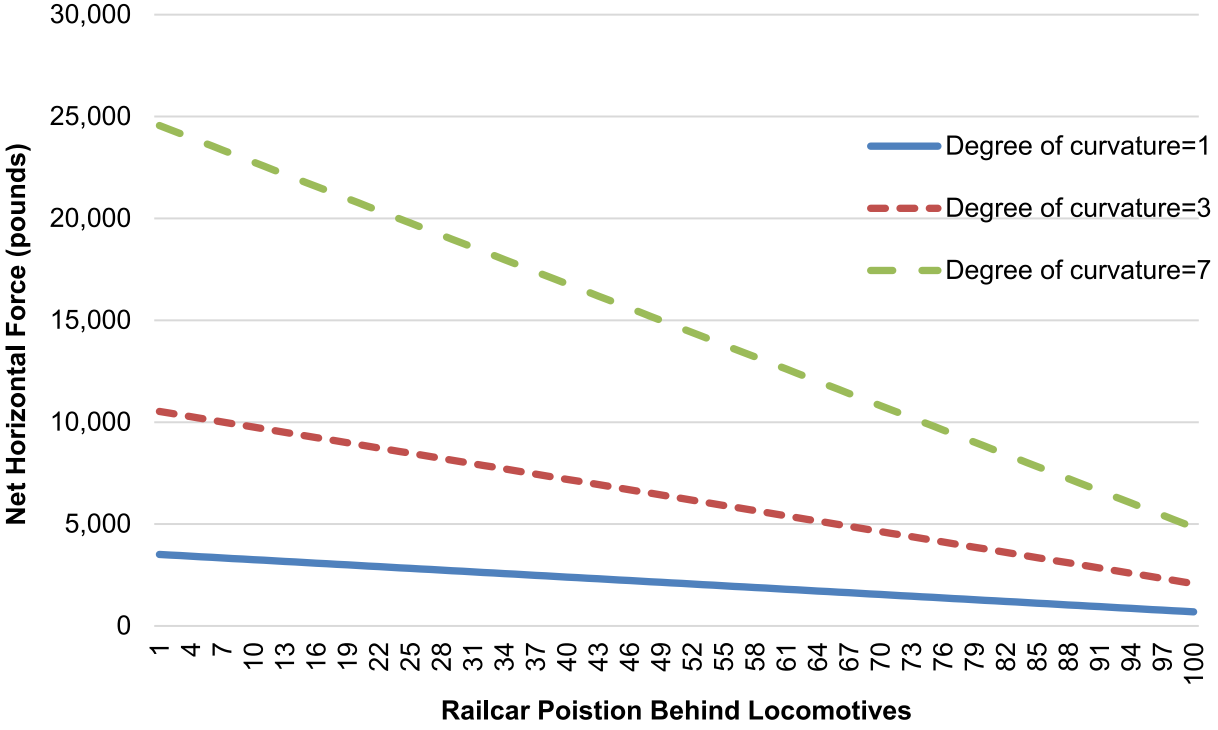


Figure 8: Variation in net horizontal forces in a 100-car freight train on a 1-percent downgrade for three different degrees of curvature

2.3.3 Equivalent Superelevation Due to In-Train Forces

Using Equations 1 and 5, the lateral component of in-train forces acting on a railcar can be further transformed into the equivalent amount of superelevation required to provide equal force on a given degree curve. The value of equivalent superelevation is independent of train speed but varies with railcar position in the train (or absolute magnitude of in-train forces), grade direction (upgrade with draft forces or downgrade with buff forces) and degree of curve (Figure 9). Positive values of equivalent superelevation on ascending grades indicate where the in-train forces cause the railcar to behave in the cant excess (overbalance) condition as if extra superelevation were installed. Negative values of equivalent superelevation on descending grades indicate where the in-train forces cause the railcar to behave in the cant deficiency (underbalance) condition as if less superelevation were installed than actually is. The values in Figure 9 were calculated for a 100-car train of loaded 286,000-lb railcars. It was assumed that all locomotives are at the front of the train and the entire train is on a 1-percent ascending grade or 1-percent descending grade.

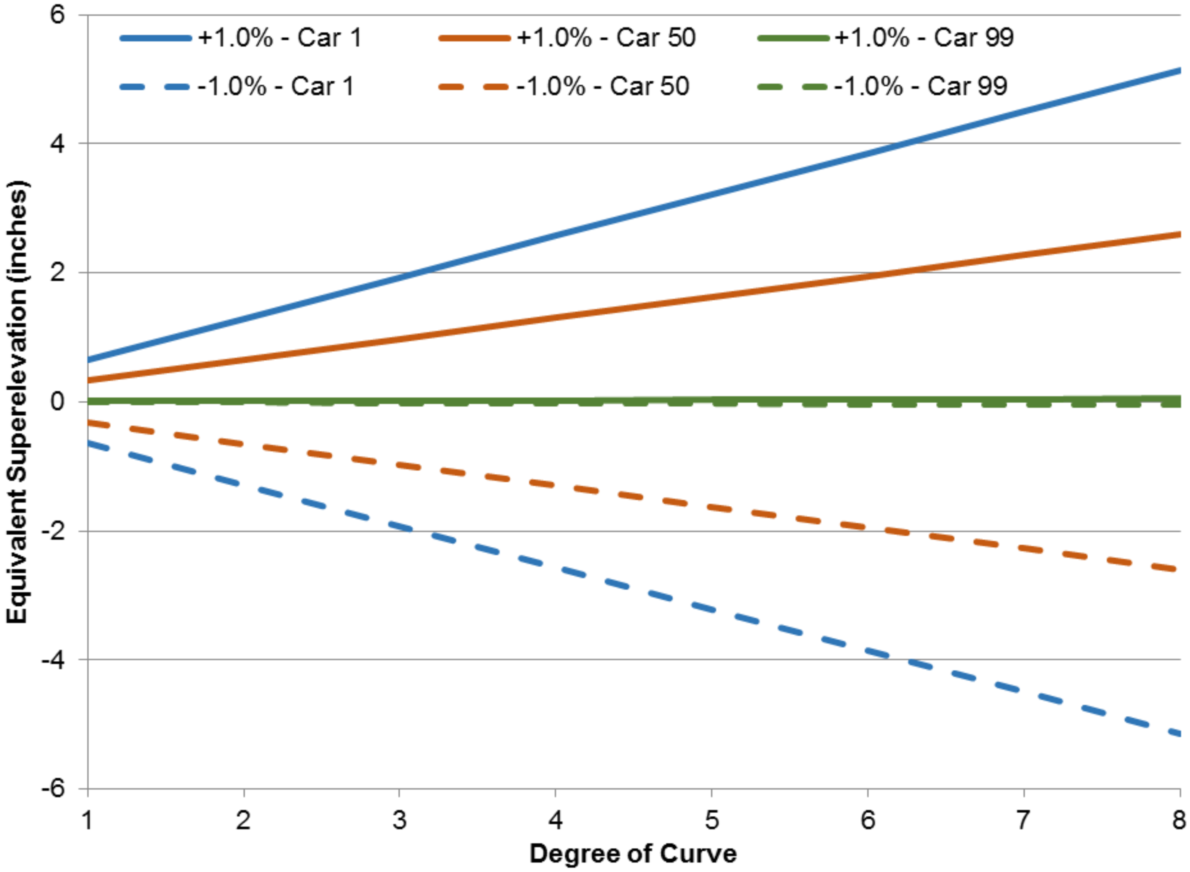


Figure 9: Equivalent superelevation due to in-train forces at different positions in a 100-car train on a 1-percent downgrade or upgrade

As described above, equivalent superelevation takes its extreme values for the first railcar in the train. On an 8-degree curve, depending on the direction of the grade, just over 5 inches of superelevation must be added or subtracted to restore the quasi-static curving conditions for the first railcar. The effect decreases as the degree of curve decreases. The final railcars in the 100-car train do not experience large enough in-train forces to alter superelevation requirements.

The form of Figure 9 suggests two complications to superelevation design on grades. The first is that within an individual train, each railcar may have its own equilibrium superelevation. This makes it nearly impossible to achieve a consistent value of cant deficiency for the entire train. The second complication is that upgrade and downgrade trains are likely to have very different equilibrium superelevation requirements. For a 100-car train negotiating a 8-degree curve on a 1-percent grade, the first railcar in a downgrade train may require 10 inches more superelevation (combined actual and unbalanced) than the first railcar in an upgrade train. It is unlikely that a single superelevation design can serve these trains equally well, even if they are operating at similar speed.

To address this difficulty, software tools have been developed to assist with design of curve superelevation on steep grades where many different types of loaded and empty trains operate (Roney, 2009). Roney et al. (2010) has also shown that the effective placement of distributed locomotives throughout the train and optimized superelevation provide gains in the areas of train speed and lateral loads on curves in mountainous terrain. Oldknow and Eadie (2010) demonstrated through experiments on two heavy-haul freight rail lines that conditions on the ascending grade do not solely dictate the superelevation and speed requirements; descending grades and other factors must also be considered.

2.4 Maintenance and Safety Considerations

Curve superelevation policies should be based on the actual speeds that a train can attain. In developing policy, care must be taken to include the effects of load eccentricity such that during a sudden startup of a train stopped on a curve, the train does not roll over to the inside of the curve due to in-train forces. At the same time, policies must provide adequate elevation to ensure trains do not roll over toward the high side of the curve or allow wheel climb to occur on the outside rail at speed. Policies must also consider wind forces, ensuring that either a stationary or moving train will not be blown off the track in open terrain.

Together, these conditions create an envelope of safe operating speeds for a given superelevated curve. However, operating at certain boundary conditions that are safe from a rail vehicle dynamics perspective can increase the stress state of the track infrastructure on curves. The long-term maintenance cost implications of these conditions, and potential derailment risk from track defects that develop slowly over time, further narrows the desirable range of curving speeds for a certain amount of actual superelevation on a given degree of curvature.

Increasing vertical wheel loads above the static value observed on tangent track (ignoring other dynamic effects) and introducing additional lateral forces at the wheel/rail interface has maintenance implications for curves. Increased forces due to the overbalance condition increase vertical wear on the low rail (Sadeghi and Akbari, 2006). The overbalance condition has also been shown to increase the frequency of track maintenance on curves and promote the development of rolling contact fatigue (Tournay et al., 2014a-c). Union Pacific Railroad concluded that correcting improperly superelevated track to eliminate overbalance conditions is the most significant factor in reducing excessive flange wear on the high and low rail on curves (Igwemezie at al., 2006). Sadeghi and Shoja (2013) examined the deterioration of crossties due to cant deficiency and concluded the underbalance condition showed less wear compared to overbalance. The overbalance condition may also result in increased derailment risk due to "string-lining" or rail roll-over.

Tournay et al. (2014a-c) suggested the overbalance condition should be avoided if possible. Since in-train forces on grades and variation in train speeds can produce overbalance conditions, some margin is needed between the design speed and the equilibrium speed to avoid cant excess. For this reason, cant deficiency is preferred to excess cant (Tournay et al., 2014a-c), and current design practice sets maximum track speed above the equilibrium speed by specifying a design value of cant deficiency. Cant deficiency allows higher curving speeds for a given amount of superelevation, facilitating reduced running time on existing track geometry. However, the underbalance condition may result in wear on the high rail, poor ride comfort, and increased risk of vehicle-overturning derailments due to over-speed incidents.

3. Design Guidelines and Standards

To synthesize industry knowledge and tools used to evaluate superelevation on different curve scenarios, the project team conducted an extensive review of current industry standards and best practices for setting design superelevation on curves. The results of this review are summarized in the following sections. The first section reviews the applicable Federal Railroad Administration (FRA) regulations and American Railway Engineering and Maintenance-of-Way Association (AREMA) guidelines for superelevation. The next section reviews specific design standards adopted by Class I freight railroads. The third section reviews superelevation standards adopted by passenger and commuter rail operators. The approaches to superelevation are then summarized, compared, and contrasted through several examples. To provide an international context, the final section provides a description of superelevation practice in Australia and Sweden.

3.1 FRA Regulations and AREMA Recommended Practices

At a minimum, the superelevation on railway curves must adhere to FRA Track Safety Standards. It is also common design practice to follow guidelines established by AREMA.

3.1.1 FRA Track Safety Standards

The U.S. Code of Federal Regulations Title 49 Part 213 (FRA Track Safety Standards) Subpart C §213.57 and Subpart G §213.329 prescribe that the outside rail on a curve may not be more than 8 inches above the inside rail on Track Classes 1 and 2, and 7 inches on Track Classes 3 through 9. Further, the outside rail of a curve may not be lower than the inside rail except when engineered to address specific track or operating conditions. The limits in §213.63 and §213.331 apply in all cases. These two standards effectively place upper and lower bounds on superelevation for railway curves in the U.S. (It should be noted that FRA Track Safety Standards define the limits, which if exceeded, compromise the safe operation of trains. Accordingly they **must not** be considered as design standards, nor should they be used in that manner.)

The maximum operating speed on railway curves (Equation 4) is also prescribed in FRA Track Safety Standards Subpart C §213.57 and Subpart G §213.329.

By FRA regulation, all rolling stock types are allowed to operate though curves at speeds that generate up to 3 inches of cant deficiency. Some rolling stock may operate though curves at speeds that generate greater than 3 inches of cant deficiency, provided they meet the regulatory requirements of Subpart C §213.57(d-j) or Subpart G §213.329(d-h). FRA regulations do not specify an upper limit on the amount of cant deficiency that can be approved. The current maximum value approved for revenue service operation in the U.S. is for Amtrak's Acela Express service at 7 inches cant deficiency. Traffic operating at a greater speed than the predominant traffic can take advantage of these higher allowable cant deficiency limits in order to operate at higher speeds around curves.

3.1.2 AREMA Recommended Practices

Chapter 5, Part 3, Section 3.3 of the AREMA Manual for Railway Engineering (AREMA, 2015) provides additional background on the derivation of Equations 1-5 for establishing equilibrium superelevation conditions and maximum speed at a given amount of cant deficiency. Chapter 17, Part 3, Section 3.2.5 provides additional information related to cant deficiency considerations for mixed passenger and freight operations.

The AREMA sections indicate it is impossible to set a single equilibrium superelevation for all classes of traffic when trains move at different speeds. Slow trains will increase wear on the inside rail, while higher-speed trains cause additional wear on the outside rail. The manual does not provide any additional information on how to establish superelevation to manage these concerns. However, the manual does indicate that conventional types of passenger equipment can safely and comfortably negotiate curves at speeds generating up to 3 inches of cant deficiency, and that certain passenger equipment trains with specially designed components can operate in excess of 3 inches of cant deficiency.

Additional information from the AREMA Practical Guide for Railway Engineering (AREMA, 2003) is as follows:

- Superelevation should be applied in ¼-inch increments.
- Even if Equation 1 indicates no superelevation is required for a curve operated at a given speed, a minimum amount of superelevation (approximately ½ inch to ¾ inch) should be installed to allow for track geometry deviations.
- Individual railway standards for superelevation and cant deficiency supersede any general recommendations from AREMA.

3.2 Class I Freight Railway Standards

Each Class I freight railway has its own design standards for superelevation that are summarized in the following sections.

3.2.1 BNSF Railway

The BNSF Railway Engineering Instructions (BNSF, 2000) describe the procedure for establishing curve superelevation using Equation 5:

- Required superelevation for maximum passenger train speed is calculated with 3 inches of cant deficiency.
- Required superelevation for maximum freight train speed is calculated with 2 inches of cant deficiency.
- On curves where the required superelevation is different between freight and passenger trains, use the greater superelevation.

Additional BNSF requirements are as follows:

- Minimum superelevation on curves is ¾ inch.
- Superelevation cannot exceed FRA limits.
- Maximum actual superelevation is 5 inches but a maximum of 4 inches is desirable.
- Superelevation is maintained to the nearest ⅛ inch.

Although not explicitly described in the standards, BNSF allows some passenger equipment to operate at cant deficiency greater than 3 inches. For example, Talgo equipment operates on BNSF in the Pacific Northwest at up to 5 inches of cant deficiency.

3.2.2 Canadian Pacific Railway

Canadian Pacific (CP) Railway Standard Practice Circular 02 describes the procedure for establishing curve superelevation using Equation 3:

- Required superelevation for maximum passenger train speed is calculated with 3 inches of cant deficiency.
- Required superelevation for maximum freight train speed is calculated with 2 inches of cant deficiency.
- On curves where the required superelevation is different between freight and passenger trains, use the greater superelevation.

Steep grades on many CP routes cause wide differences in train operating speeds and introduce lateral components of drawbar forces on curves. To account for these effects, CP has partnered with Advanced Rail Research Technologies to better select values for actual superelevation and cant deficiency on individual curves. The Advanced Superelevation Toolkit (ASET) software developed through this partnership determines the bandwidth of superelevation required for all train types traversing a particular curve. The software then selects an appropriate actual superelevation value to minimize track forces.

3.2.3 CN Railway

CN Recommended Method 1305 (CN, 2002) describes the procedure for establishing curve superelevation using Equations 2 and 3:

- Range of superelevation for maximum passenger train speed is calculated with 3 inches of cant deficiency (minimum E_a) and equilibrium (maximum E_a).
- Range of superelevation for maximum freight train speed is calculated with 2 inches of cant deficiency (minimum E_a) and equilibrium (maximum E_a).
- Range of superelevation for maximum specialized passenger train speed is calculated with an appropriate level of cant deficiency (minimum E_a) and equilibrium (maximum E_a). For LRC equipment, E_u is 6 inches.
- On curves where the required superelevation is different between freight and passenger trains, an acceptable range of superelevation is defined by:
  - The smallest maximum E_a is the largest value of superelevation that can be installed without creating an overbalance condition.
  - The largest minimum E_a is the smallest value of superelevation that can be installed while still meeting desired track speed for each type of train.
- The actual superelevation can be set anywhere within this range. However, since the traffic distribution tends to consist of a large portion of slow and heavy freight trains and fewer passenger or high-speed freight trains, CN recommends installing the minimum superelevation within this range.

The CN procedure describes how greater superelevation will result in the weight of slower trains being transferred to the low rail causing damage to the low rail. CN also states that using a lesser superelevation will result in faster trains producing greater lateral forces through the curve. This can increase gauge widening and gauge face wear.

The CN procedure also states that a speed zone analysis shall be performed to determine appropriate speeds for use in the superelevation calculation. For example, where a 60-mph freight speed requires the maximum-allowable 5 inches of superelevation (calculated at equilibrium), this should only be installed where the majority of trains can achieve 60 mph. If very few trains can achieve 60 mph, and most travel less than 50 mph, a lesser amount of superelevation should be installed (as calculated for 60 mph with 2 inches of cant deficiency).

Additional CN requirements are as follows:

- Minimum superelevation on curves is ½ inch.
- Maximum actual superelevation is 5 inches.
- Superelevation is maintained to the nearest ⅛ inch.

3.2.4 CSX Transportation

CSX Standard Plan 2511 (CSX, 2002) provides instructions for establishing curve superelevation using Equation 5:

- Superelevation for maximum freight train speeds up to and including 60 mph is calculated with 1½ inches of cant deficiency.
- Superelevation for maximum freight train speeds of 65 mph and 70 mph is calculated with 2½ inches of cant deficiency.
- Actual superelevation is determined by the above freight calculations. Maximum speeds for passenger trains are calculated by Equation 4 with the actual superelevation E_a from the freight calculation and the appropriate cant deficiency E_u for the type of passenger service. The amount of cant deficiency may range from 3 to 5 inches, depending on the type of passenger equipment and FRA requirements.

Additional CSX requirements are as follows:

- Minimum superelevation on curves is ½ inch.
- Maximum actual superelevation is 5 inches on any curve.
- Maximum superelevation on curves greater than 3 degrees is 4½ inches.
- Maximum superelevation is 4 inches on grades where trains regularly operate below 25 mph.
- Superelevation is maintained to the nearest ¼ inch.
- Curves shall be regularly examined for premature or accelerated wear on the high or low rail. A request for deviation from the standard superelevation can be submitted in these situations.

3.2.5 Norfolk Southern Railway

The Norfolk Southern (NS) Public Projects Manual Drawing No. 7 (Norfolk Southern, 2013) provides instructions for establishing curve superelevation using a table. Although not explicitly indicated in the standard, the table closely follows the following calculations with Equation 5 and allowances for rounding to the ½ inch:

- Superelevation for maximum passenger train speed is calculated with 3 inches of cant deficiency.
- Superelevation for maximum freight train speed is calculated with 2 inches of cant deficiency.
- On curves where passenger and freight trains both operate, the higher superelevation is to be used.

The NS standard also states the proper elevation of curves requires consideration of degree of curvature, type of traffic, location, grade, speed, and local conditions.

Additional NS requirements are as follows:

- Maximum superelevation shall not exceed 4 inches for freight trains.
- Maximum superelevation shall not exceed 5 inches for passenger trains.

3.2.6 Union Pacific Railroad

The Union Pacific (UP) Railroad Engineering Standards (Union Pacific, 2011) describe the procedure for establishing curve superelevation using Equation 5:

- Superelevation for maximum passenger train speed is calculated with 3 inches of cant deficiency.
- Superelevation for maximum freight train speed is calculated with 1 inch of cant deficiency if the calculated superelevation is 4 inches or less. If the calculation indicates superelevation in excess of 4 inches is required, recalculate with 2 inches of cant deficiency.

The UP standard does not indicate how differences between passenger and freight superelevation are resolved. However, the standards indicate that the chief engineer can approve deviations from the standard, provided speeds do not exceed those calculated with 3 inches of cant deficiency as prescribed by FRA.

Additional UP requirements are as follows:

- Minimum superelevation on curves is ¼ inch.
- Maximum actual superelevation is 5 inches but a maximum of 4 inches is desirable.
- Superelevation is maintained to the nearest ¼ inch.

3.3 Passenger and Commuter Standards

Many passenger and commuter rail operators that own track have their own design standards for superelevation that are summarized in the following sections. While several of these were inherited from freight railroads, some have been modified to better accommodate the operational objectives of the passenger operator.

3.3.1 National Railroad Passenger Corporation (Amtrak)

Amtrak Specification Number 63 (National Railroad Passenger Corporation, 2015) describes procedures for determining superelevation on the Northeast Corridor (NEC) between New Haven, CT and Boston, MA; and between New Rochelle, NY and Washington, DC. (Note: Metro-North Commuter Railroad operates and maintains the NEC between New Haven, CT and New Rochelle, NY and uses their own standards for defining curve geometry.) Superelevation requirements are calculated with Equation 5 as follows:

- Superelevation for maximum passenger train speed is calculated with 3 to 7 inches of cant deficiency, depending on the passenger equipment being used:
    - Acela with tilt active is allowed 7 inches of cant deficiency.
    - Acela with tilt disabled, AEM7, HHP, P32, P40, P42, Amfleet, and Horizon equipment is allowed 5 inches of cant deficiency.
    - Other passenger cars (non-mail and express) are allowed 4 inches of cant deficiency.
    - Mail and express cars are allowed 3 inches of cant deficiency.
- Superelevation for maximum freight train speed is calculated with 1½ inches of cant deficiency.

Amtrak standards specify that superelevation greater than 4 inches should be avoided wherever freight trains are operated. Superelevation in these areas should be limited by increasing the cant deficiency where possible.

Additional Amtrak requirements are as follows:

- Minimum superelevation on curves is ½ inch, with some exceptions.
- Maximum actual superelevation is 5½ inches.
- Curves on open-deck bridges and through grade crossings should not be operated with more than 5 inches of cant deficiency.

A final requirement of Amtrak is the superelevation design must not use the maximum limits of both superelevation and underbalance in combination in the same curve. The actual superelevation (E_a) added to the cant deficiency (E_u) must be at least 1 inch less than the sum of the maximum allowable superelevation and the maximum allowable cant deficiency for the type of equipment specified. For example, if P42 locomotives and Amfleet railcars (maximum cant deficiency of 5 inches) are operating on a route with a maximum actual superelevation of 5½ inches, the maximum possible Ea + Eu is 9½ inches (5 plus 5½ minus 1 inch) for the calculation of maximum speed with Equation 4. This requirement is introduced to allow for track cross-level or alignment deficiencies that would cause slow orders if maximum passenger train speeds were set according to both maximum actual superelevation and maximum allowable cant deficiency.

3.3.2 California High-Speed Rail Project Shared-Corridor Standards

California High-Speed Train Project Technical Memorandum TM 1.1.6 “Alignment Standards for Shared-Use Corridors” describes standards and approaches for establishing superelevation to support the operation of higher-speed trains on the same track infrastructure as conventional freight and passenger trains (California High-Speed Rail Authority, 2008). The design value of superelevation is influenced by:

- Maximum speed limit
- Calculated normal and maximum speeds of higher-speed trains
- Calculated normal and maximum speeds of other passenger trains
- Calculated normal and maximum speeds of freight trains where applicable

Normal speed is defined as 90 percent of the maximum speed in these standards. The document recognizes there may be multiple values of freight train speeds, depending on the nature of the traffic. It is also noted that high values of superelevation cause passenger discomfort and maintenance difficulties when trains are operated at lower speeds.

Maximum actual superelevation on lines with combinations of higher-speed and other passenger trains (but no freight trains) are set as follows:

- Desirable is 4 inches (or 3 inches where some trains will have lower speeds)
- Limiting is 5 inches (or 4 inches where some trains will have lower speeds)
- Exceptional is 6 inches (or 4½ inches where some trains will have lower speeds)

The following criteria is specified for cant deficiency when calculating maximum curve speeds using Equation 4:

- Minimum cant deficiency is 1 inch to avoid truck hunting issues when trains operate at equilibrium (or balanced) speed for the actual superelevation through curves.
- Desired maximum cant deficiency is 3 inches.
- Limiting/exceptional maximum cant deficiency is 4 inches by FRA regulation.

The California HSR standards dictate the actual superelevation shall be set to provide the best practical ride quality to the majority of the passengers on the trains passing over the particular curve without violating criteria limits. To achieve this goal, additional requirements are specified as follows:

- Applied superelevation shall be 1 inch less than that necessary to balance the normal (not maximum) speed. This avoids the situation of trains at the normal speed encountering overbalance conditions.
- In locations where multiple types of traffic operate, the appropriate applied superelevation shall first be calculated for each type of traffic. Then a value based on the proportions of each type of traffic shall be developed. This value should become the designated applied superelevation unless it results in violation of the desirable limit of cant deficiency for the fastest train’s normal speed or violation of the limiting unbalance

for the sections speed limit. Should either violation occur, the applied superelevation should be increased sufficiently to meet these limits.

- While applied superelevation in excess of that required to balance any train being operated is undesirable, it may be necessary for this situation to occur to avoid restricting the speed of high-speed trains.

3.3.3 Caltrain Commuter Rail

The Caltrain Commuter Rail Design Criteria provides instructions for establishing curve superelevation using Equation 5:

- Superelevation for maximum passenger train speed is calculated with 3 inches of cant deficiency.

No information is provided on freight trains or superelevation for variable train' speeds.

Additional Caltrain commuter rail requirements are as follows:

- Minimum superelevation on curves is ½-inch.
- Maximum actual superelevation is 5 inches.
- Superelevation is maintained to the nearest ¼ inch.

3.3.4 Massachusetts Bay Transportation Authority

The Massachusetts Bay Transportation Authority (MBTA) Commuter Rail Design Standards Manual (MBTA, 1996) provides instructions for establishing curve superelevation:

- Superelevation for maximum train speed is calculated with 1½ inches of cant deficiency (preferred) or 2¾ inches of cant deficiency (maximum).

Additional MBTA commuter rail requirements are as follows:

- Minimum superelevation on curves is 1 inch
- Maximum actual superelevation is 6 inches.
- It is desirable to limit actual superelevation to 4 inches on routes where through freights operate and where trains are likely to stop or operate below the maximum design speed on a regular basis.
- Within stations it is desirable to limit actual superelevation to 3 inches and use 2¾ inches of cant deficiency to allow express operation at maximum authorized speed.

3.3.5 Southern California Regional Rail Authority (Metrolink)

The Southern California Regional Rail Authority (Metrolink) Design Criteria Manual (Southern California Regional Rail Authority, 2014) provides instructions for establishing curve superelevation:

- Superelevation for maximum passenger train speed is calculated with 3½ inches of cant deficiency.

- Superelevation for maximum freight train speed is calculated with 2 inches of cant deficiency.
- Actual superelevation is set according to passenger requirements but is then checked to ensure the maximum freight trains speed corresponds to 1 to 2 inches of cant deficiency.

Additional Metrolink commuter rail requirements are as follows:

- Maximum actual superelevation is 5 inches.

3.3.6 *Utah Transit Authority*

The Utah Transit Authority Commuter Rail Design Criteria (Utah Transit Authority, 2010) sets requirements for superelevation on curves. Superelevation is calculated using Equations 2 and 3 and the following parameters:

- Maximum cant deficiency is 3 inches
- Maximum actual superelevation is 5 inches with some exceptions (described below).
- Equilibrium superelevation shall meet the design speed and be divided as closely as possible between ⅔ actual superelevation and ⅓ cant deficiency. For example, if using Equation 4 the combination degree of curve and design speed requires an equilibrium superelevation of 6 inches, then 4 inches of actual superelevation should be installed and trains should operate with 2 inches of cant deficiency.

In areas where vehicles will frequently operate at lower speeds, actual superelevation is limited to 4 inches. Station approaches are generally designed to match the speed of the corresponding turnout, potentially decreasing superelevation requirements on adjacent curves.

Additional Utah commuter rail requirements are as follows:

- Minimum superelevation on curves is 1 inch.
- Actual superelevation is maintained to the ½ inch.

3.3.7 *Other Operators*

The Metro-North Railroad criteria for establishing the maximum allowable speed on curves are detailed in MW 4, Manual for Construction, Maintenance and Inspection of Track. Metro-North Railroad's preferred maximum actual superelevation is 4 inches at 1½ inches of cant deficiency.

The Northeast Illinois Regional Commuter Railroad Corporation (Metra) designs curve superelevation with a maximum of 2-1/2 inches of cant deficiency.

The Southeastern Pennsylvania Transportation Authority (SEPTA) Railroad Division SMW-100 Track Maintenance and Construction Standards provides limited information on superelevation design. The standards state that the Manager of Track Engineering shall establish the amount of elevation, underbalance, and speed to be placed and maintained on each curve.

3.4 Summary and Comparisons of U.S. Practice

Comparison of superelevation design practices (Table 1) reveal that no two operators use the exact same criteria. With the exception of MBTA, all operators calculate superelevation for passenger operations with at least 3 inches of cant deficiency. Several operators allow passenger train curve speeds to be established for higher levels of cant deficiency where equipment meets FRA requirements. Such provisions are more commonly documented in the design criteria of the passenger operators than the freight railways. Freight railways may not explicitly address operation with higher levels of cant deficiency in their design standards and instead elect to handle them on a case-by-case basis. An example of current operations that are an exception to published design criteria is the operation of Talgo equipment on BNSF, with 5 inches of cant deficiency.

Table 1. Summary of current US superelevation design criteria

Railway or Operator	Cant Deficiency = E_u (inches)		Superelevation - E_a (inches)			Type of Train Governing E_a
	Freight	Passenger	Minimum	Maximum	Increment	
BNSF	2.00	3.00	0.75	5.00	0.125	Largest E_a of freight or passenger
CP	2.00	3.00	N/A	N/A	N/A	Selected with software tool
CN	2.00	3.00 or 6.00	0.50	5.00	0.125	Minimum E_a in range of operating speeds
CSX	1.50 or 2.50	3.00 or 4.00	0.50	5.00	0.25	Freight
NS	2.00	3.00	1.00	5.00	0.50	Largest E_a of freight or passenger
UP	1.00	3.00	0.25	5.00	0.25	Not indicated
AMTRAK	1.50	4.00 to 7.00	0.50	5.50	N/A	Passenger
CAHSR	1.00	3.00 to 4.00	N/A	3.00 to 6.00	N/A	Proportional to support passenger speeds
CALTRAIN	N/A	3.00	0.50	5.00	0.25	Passenger
MBTA	N/A	2.75	1.00	6.00	N/A	Passenger
METROLINK	2.00	3.50	N/A	5.00	N/A	Passenger but check freight cant deficiency
UTAH	N/A	3.00	1.00	5.00	0.50	Passenger

N/A = information not available

Consistent with the findings of the research undertaken, the design criteria information provided in this report is accurate, complete, and current per the date of report publishing.

UP uses the lowest value of cant deficiency for freight operations at 1 inch. This relatively low value of cant deficiency results in a smaller difference in speed between the design and equilibrium conditions than is observed with other design criteria. Thus UP has the smallest speed range for freight trains to operate below track speed but still avoid the overbalance condition that is frequently cited as a cause of maintenance issues on the low rail. However, the low value of cant deficiency tends to result in more actual superelevation being installed for a given freight speed compared to other criteria. This potentially benefits passenger operators on UP routes, as there can be a larger difference between freight and passenger train speeds compared to the design criteria on other freight railways.

While the freight railways all allow up to 5 inches of actual superelevation, most state a preference for less than 4 inches of actual superelevation. The passenger operators are more likely to allow superelevation in excess of 5 inches. However, many state a preference that less than 4 inches of actual superelevation be installed where freight trains also operate.

The CN, California HSR, and Metrolink design criteria provide the most extensive procedures for consideration of differences in speed between various types of trains. Interestingly, Amtrak does not describe these provisions. This is likely due to their standards being developed for Amtrak-owned track in the Northeast Corridor that is dominated by passenger operations and subject to little through freight traffic.

CSX is the only freight operator that explicitly states that superelevation is designed for freight train speeds and passenger train speeds are set according to the actual superelevation and allowable cant deficiency. Other freight operators indicate that the larger of actual superelevation required for freight or passenger train speeds shall be installed. Depending on the combination of desired freight and passenger train speeds and allowable cant deficiency, this may create a situation where slower freight trains operate on track superelevated for higher passenger speeds. In such cases the freight trains may be operating below equilibrium speed and in the overbalance condition with track maintenance implications.

The following sections show examples of some of the differences in the current practices for superelevation.

3.4.1 Specified Actual Superelevation

This example illustrates differences in calculated maximum freight and passenger train speeds for an existing curve where the actual superelevation has been specified as a certain value (likely corresponding to the current actual value in track).

The example curve is a 2-degree curve with 3 inches of actual superelevation subject to freight and passenger traffic. The passenger train was assumed to be a conventional single-level Amtrak train as commonly found in regional-intercity service. According to Equation 1, equilibrium speed on this curve is 46 mph.

The maximum allowable freight and passenger trains speeds on this curve are calculated according to each of the superelevation design criteria described previously (Table 2). Columns 2 and 3 indicate the freight and passenger cant deficiency used specifically for this example. The resulting freight speeds are rounded to the nearest 5-mph increment while passenger speeds are reported to the nearest mile-per-hour to better demonstrate the incremental benefit of increasing cant deficiency.

Maximum allowable freight speed on this curve ranges from 50 to 60 mph. On UP there is only a 4-mph difference between the maximum freight train speed and the equilibrium speed. Thus it is likely that some freight trains may experience an overbalance condition.

Maximum allowable passenger train speed ranges from 64 to 75 mph. The highest passenger train speeds are obtained where the passenger trains are allowed to operate with higher cant deficiency. In this case, the extra cant deficiency allows the passenger train speed to be increased by 5 to 10 mph compared to criteria that use the regular FRA maximum cant deficiency of 3 inches.

Table 2. Calculated speeds for 2-degree curve with 3 inches actual superelevation

Railway or Operator	Cant Deficiency = E_u (inches)		Superelevation E_a (inches)	Speed (mph)			Type of Train Governing E_a
	Freight	Passenger		Equilibrium	Freight	Passenger	
BNSF	2.00	3.00	3.00	46	60	65	
CP	2.00	3.00	3.00	46	60	65	
CN	2.00	3.00	3.00	46	60	65	
CSX	1.50	4.00	3.00	46	55	70	Passenger E_u = 4”
NS	2.00	3.00	3.00	46	55*	65	Maximum freight speed in table is 55 mph
UP	1.00	3.00	3.00	46	50	65	
AMTRAK	1.50	5.00	3.00	46	55	75	Passenger E_u = 5”
CAHSR	1.00	4.00	3.00	46	50	70	Passenger E_u = 4”
CALTRAIN	N/A	3.00	3.00	46	--	65	
MBTA	N/A	2.75	3.00	46	--	64	
METROLINK	2.00	3.50	3.00	46	60	68	
UTAH	N/A	3.00	3.00	46	--	65	

N/A = information not available

Consistent with the findings of the research undertaken, the information provided in this report is accurate, complete, and current per the date of report publishing.

3.4.2 Specified Freight Train Speed

This example illustrates differences in calculated actual superelevation and maximum passenger train speed for an existing curve where the maximum freight train speed is specified.

The example curve is a 2-degree curve with a freight train speed of 50 mph subject to freight and passenger traffic. The passenger train was assumed to be a conventional single-level Amtrak train as commonly found in regional-intercity service. Equilibrium speed on this curve varies with the amount of superelevation installed.

The actual superelevation and maximum allowable passenger train speed on this curve are calculated according to each of the superelevation design criteria described previously (Table 3). Columns 2 and 3 indicate the freight and passenger cant deficiency used specifically for this example. Passenger speeds are reported to the nearest mile-per-hour to better demonstrate the incremental benefit of increasing cant deficiency. Several commuter design criteria lack required information on freight cant deficiency to complete the design.

As dictated by the 50-mph freight speed, actual superelevation varies from 1½ inches to 2½ inches. This range corresponds directly to the range in freight cant deficiency prescribed in the superelevation design criteria.

Table 3. Calculated superelevation and passenger speed for 2-degree curve with 50-mph freight train speed

Railway or Operator	Cant Deficiency = E_u (inches)		Superelevation E_a (inches)	Speed (mph)			Type of Train Governing E_a
	Freight	Passenger		Equilibrium	Freight	Passenger	
BNSF	2.00	3.00	1.50	32	50	56	
CP	2.00	3.00	1.50	32	50	56	
CN	2.00	3.00	1.50	32	50	56	
CSX	1.50	4.00	2.00	37	50	65	Passenger E_u = 4”
NS	2.00	3.00	1.50	32	50	56	
UP	1.00	3.00	2.50	42	50	63	
AMTRAK	1.50	5.00	2.00	37	50	71	Passenger E_u = 5”
CAHSR	1.00	4.00	2.50	42	50	68	Passenger E_u = 4”
CALTRAIN	N/A	3.00	N/A	N/A	N/A	N/A	Criteria does not support freight speed
MBTA	N/A	2.75	N/A	N/A	N/A	N/A	Criteria does not support freight speed
METROLINK	2.00	3.50	1.50	32	50	56	
UTAH	N/A	3.00	N/A	N/A	N/A	N/A	Criteria does not support freight speed

N/A = information not available

Consistent with the findings of the research undertaken, the information provided in this report is accurate, complete, and current per the date of report publishing.

Equilibrium speed on this curve ranges from 32 to 42 mph. This provides an operating speed buffer of 8 to 18 mph before slower freight trains experience overbalance conditions. Maximum allowable passenger train speed ranges from 56 to 71 mph. The highest passenger train speeds are obtained where the passenger trains are allowed to operate with higher cant deficiency and the superelevation is designed for lower values of freight cant deficiency. In this case, the combination of extra passenger cant deficiency and less freight cant deficiency allows the passenger train speed to be increased by as much as 15 mph, compared to criteria that use the 3 inches and 2 inches of cant deficiency for passenger and freight trains, respectively.

3.4.3 Specified Passenger Train Speed

This example illustrates differences in calculated actual superelevation and maximum freight train speed for an existing curve where the maximum passenger train speed is specified.

The example curve is a 2-degree curve with a freight train speed of 50 mph subject to freight and passenger traffic. The passenger train was assumed to be a conventional single-level Amtrak train as commonly found in regional-intercity service. Equilibrium speed on this curve varies with the amount of superelevation installed.

The actual superelevation and maximum allowable freight train speed on this curve are calculated according to each of the superelevation design criteria described previously (Table 4). Columns 2 and 3 indicate the freight and passenger cant deficiency used specifically for this example. The resulting passenger speeds are reported to the nearest mile-per-hour to better demonstrate the incremental benefit of increasing cant deficiency. Several of the commuter design criteria lack required information on freight cant deficiency to complete the design for a specific freight speed.

Table 4. Calculated superelevation and freight speed for 2-degree curve with 75-mph passenger train speed

Railway or Operator	Cant Deficiency = E_u (inches)		Superelevation E_a (inches)	Speed (mph)			Type of Train Governing E_a
	Freight	Passenger		Equilibrium	Freight	Passenger	
BNSF	2.00	3.00	4.875	59	70	75	
CP	2.00	3.00	5.00	59	70	75	
CN	2.00	3.00	4.875	59	70	75	
CSX	1.50	4.00	4.00	53	60	75	Passenger E_u = 4”
NS	2.00	3.00	5.00	59	55*	75	Maximum freight speed in table is 55 mph
UP	1.00	3.00	5.00	59	65	75	
AMTRAK	1.50	5.00	3.00	46	55	75	Passenger E_u = 5”
CAHSR	1.00	4.00	4.00	53	55	75	Passenger E_u = 4”
CALTRAIN	N/A	3.00	5.00	59	N/A	75	Criteria does not support freight speed
MBTA	N/A	2.75	5.25	61	N/A	75	Criteria does not support freight speed
METROLINK	2.00	3.50	4.50	56	65	75	
UTAH	N/A	3.00	5.00	59	N/A	75	Criteria does not support freight speed

N/A = information not available

Consistent with the findings of the research undertaken, the information provided in this report is accurate, complete, and current per the date of report publishing.

As dictated by the 75-mph passenger speed, actual superelevation is generally around 5 inches but is as low as 3 inches for Amtrak and as large as 5¼ inches for MBTA. This observed range corresponds directly to the range in passenger cant deficiency prescribed in the superelevation design criteria. The upper end of this rage corresponds to the 5-inch maximum actual superelevation prescribed by most operators.

Equilibrium speed on this curve ranges from 53 to 61 mph, with most railway criteria yielding an equilibrium speed of 59 mph. Since a large number of freight trains travel at speeds below 59 mph, a large number of these slower freight trains will experience overbalance conditions. The operators with the lowest equilibrium speed, and hence largest tolerance for slower freight trains, are those that allow the largest passenger train cant deficiency.

Maximum allowable freight train speed ranges from 55 to 70 mph. The lowest freight train speeds are obtained where the passenger trains are allowed to operate with higher cant deficiency and the freight trains are allowed relatively small levels of cant deficiency. In this case, the combination of extra passenger cant deficiency and less freight cant deficiency allows the passenger train speed to be as much as 20 mph higher than the freight train speed. A 20-mph range gives slower trains a larger speed buffer before they travel slow enough to encounter overbalance conditions compared to other superelevation design criteria. NS is a particularly interesting case because the maximum freight train speed of 55 mph falls below the equilibrium speed of 59 mph. This implies that all freight trains would be operating in the overbalance condition. It is unlikely that such a superelevation design would be approved for lines with a predominance of freight traffic.

3.5 International Practice

To provide a context for domestic superelevation practice, two superelevation design criteria for countries with similar shared-corridor conditions were examined. The following sections summarize superelevation practices in Australia and Sweden.

3.5.1 Australia

The Australian Rail Track Corporation Limited Code of Practice, Section 5: Track Geometry provides information for design of superelevation on railway curves on the standard gage network in Australia (Australian Rail Track Corporation, 2015). Curve superelevation is designed using Equation 5 and the following parameters:

- Superelevation for maximum speed of advanced XPT passenger trains is calculated with 4¼ inches of cant deficiency.
- Maximum train speed on interstate shared corridors calculated with 3¼ inches of cant deficiency.
- Maximum train speed on heavy-haul freight lines is calculated with 3 inches of cant deficiency.

Australian standards establish cant deficiency for superelevation design by the type of corridor and typical range of operating speeds associated with that corridor. This is in contrast to U.S. practice, where different train types are assigned different cant deficiencies and the designer must reconcile differences in calculated superelevation.

Additional Australian requirements are as follows:

- Maximum actual superelevation is 6 inches on interstate shared corridors.
- Maximum actual superelevation is 5½ inches on heavy haul freight corridors.
- Actual superelevation is limited to 4¼ inches in the vicinity of stations where trains are likely to travel below maximum route speed.

3.5.2 Sweden

The Swedish Transport Administration provides information for design of superelevation on railway curves in Sweden (Lindahl, 2001). Curve superelevation is designed using the following parameters for cant deficiency:

- Superelevation for maximum speed of freight trains and conventional passenger trains with older running gear is calculated with 4 inches of cant deficiency.
- Superelevation for maximum speed of conventional passenger trains with improved running gear is calculated with 6 inches of cant deficiency where approved.

- Superelevation for maximum speed of passenger trains with improved running gear and carbody tilt systems is calculated with 9½ inches of cant deficiency.

In addition the maximum actual superelevation is 6 inches.

When calculating the actual superelevation, design speeds are increased by 30 percent to provide a margin for ride comfort and to account for increased speed in the future. This may partially explain the rationale for the allowable cant deficiency of most train types exceeding their corresponding values in U.S. practice. The higher allowable value for cant deficiency (9½ inches) is also a direct result of the use of radially steering bogies which in turn lowers the curving forces at the wheel/rail interface.

3.6 Other Practical Constraints

3.6.1 Spiral length

Spiral (transition or easement) curves are defined as transition curves with a constantly decreasing or increasing radius proportional between either a tangent and a curve (simple spiral) or between two curves with different radii (compound/intermediate spiral). More specifically, the spiral is a curve whose degree-of-curve increases directly as the distance along the curve from the point of spiral. They provide a gradual change of curvature and allow lateral acceleration, thereby improving ride comfort as the vehicle transitions from the tangent to full curvature or vice versa.

A spiral transition is also provided between circular curves and horizontal tangents as a means to develop superelevation from the level condition on tangent track to the fully superelevated condition in the circular curve. The rate at which superelevation is introduced into the track within the spiral is termed the runoff rate. To maintain ride comfort and safe operating conditions, the design criteria establish the maximum amount of change in superelevation per unit of length. The change in superelevation is typically referenced per 31 or 62 feet of track length. The maximum allowable runoff rate is specified in the FRA Track Safety Standards Subpart C §213.59 and §213.63 for Track Class 1 to 5 and in Subpart G §213.331 for Track Class 5 to 9. Thus, for a given spiral length, the maximum runoff rate also sets a maximum amount of actual superelevation that can be installed on a curve without altering horizontal geometry.

As noted previously, FRA Track Safety Standards define the limits, which if exceeded, compromise the safe operation of trains. Accordingly, they **must not** be considered as design standards, nor should they be used in that manner.

When additional actual superelevation in proposed to increase operating speed on a particular curve, the design must be checked against the maximum allowable superelevation runoff rate for the current spiral length. If the maximum runoff rate is exceeded, the proposed superelevation is infeasible without lengthening the spiral length by shifting the horizontal track geometry of the curve. Depending on horizontal clearance constraints, adjacent track centers, bridges, culverts, and overhead catenary, it may not be possible to make the geometric adjustments required to extend the spirals. In this manner, existing spiral length may constrain actual superelevation and maximum train operating speeds for a given level of cant deficiency.

3.6.2 Ride comfort

The level of ride comfort is determined by the motions of the passenger within the rail vehicle as it moves along the track (Vermeil, 2000). The level of comfort a passenger experiences as a railway vehicle negotiates a horizontal curve is in turn driven by the rate of change of lateral acceleration through the transition spiral (commonly referred to as the “jerk rate”) and by the steady-state lateral acceleration through the full body of the curve (the portion with a constant radius). This is directly related to the spiral curve length, rate of superelevation runoff, degree of curvature, amount of superelevation, and vehicle speed.

Förstberg et al. (1998) indicated that increased train speed can be achieved by using tilting trains that decrease the lateral acceleration experienced by passengers on curves. This approach allows for greater cant deficiency and trains to run typically 25 to 30 percent faster on existing curved track while maintaining good ride comfort.

Harris et al. (1998) demonstrated that the curving speed of a tilting train is limited not by passenger comfort but by the degree of weight distribution across the axles and by lateral track forces. In a well-designed system, passenger comfort is controlled by the tilt mechanism and is no longer at issue. The position of the tilt center has a significant influence on this weight transfer. Height of the tilt center, optimally at navel level for passenger comfort, determines the lateral displacement of the center of gravity of the vehicle during curving. This influences the weight distribution across the axles, affecting an upper limiting factor on curving speed. Raising this upper limit increases the cost benefit and viability of the application of tilting vehicles to a given route. The cost benefit and viability of tilt depends on a number of primary factors, including existing cant, existing track strength, design and appropriate use of transition curves, the value of decreased journey time, and frequency of curves. Secondary factors such as height of the tilt center, fuel savings and increased rail wear may also require consideration.

4. Selection of Superelevation for Multiple Train Speeds

For a given combination of degree of curve and actual superelevation, there is a single unique train speed for which equilibrium conditions exist. Any trains that operate over the route at different speeds will traverse the curve with varying amounts of cant deficiency (positive E_u) or cant excess (negative E_u). The track design engineer is faced with the task of selecting an appropriate train speed for superelevation design. It is common practice to design superelevation for the maximum allowable track speed. However, there are a number of factors that complicate this task, as described in the following sections.

An empirical approach to handling a range of train speeds is to set the superelevation on the curve to the minimum required to achieve the desired passenger speed and then to observe the rail wear over a period of time. The superelevation is then gradually increased until a desirable even wear pattern is observed on both the high and low rails. Plotkin (1997) analyzed mixed-use rail corridors and concluded that setting a common superelevation is quite complex and may create overbalance conditions for freight trains. Tournay et al. (2014a-c) concluded that superelevation for the speed of prevailing tonnage is the most desired condition.

4.1 Difficulties in Handling Multiple Train Speeds

When multiple train types operate on a rail corridor, several maximum train speeds may be specified in the timetable. When these maximum train speeds cover a range of values, selecting one to govern the superelevation design is non-trivial (Kollmar, 2006). While most freight trains operate between 40 and 60 mph, an increasing number of passenger trains are qualified by FRA to operate at higher cant deficiencies and at speeds from 70 to 150 mph on curves (FRA, 2015). While conventional 60- and 70-mph passenger train speeds largely overlapped (or were contiguous with) those of freight trains, on mixed-use corridors with higher-speed passenger trains, the distribution of train speeds is distinctly bi-modal, an example of which is shown in Figure 10.

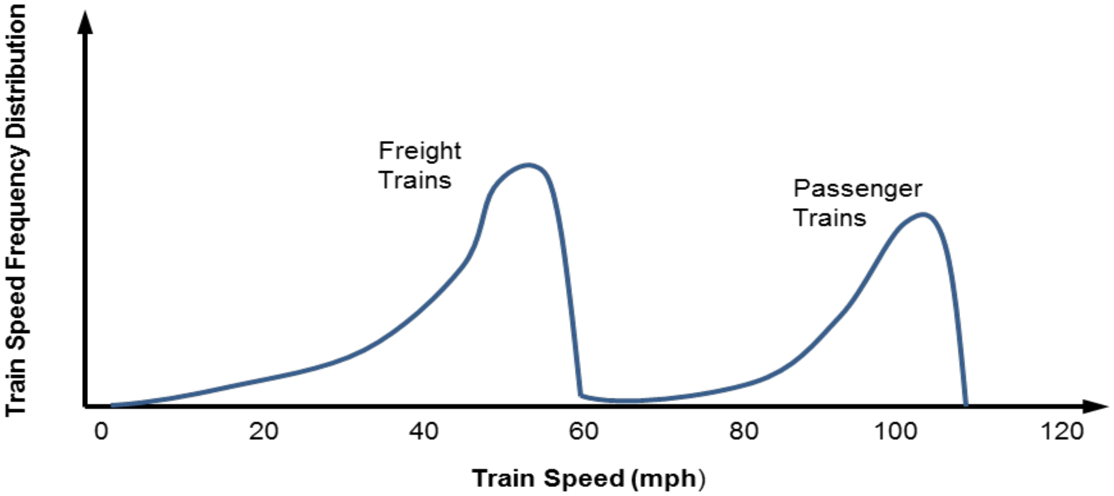


Figure 10: Distribution of train speeds on mixed-use freight and passenger corridor

In Figure 10, the majority of passenger trains are travelling much faster than the maximum freight train speed. Over 60 mph can separate the most frequent freight and passenger train speeds. As will be demonstrated later in this section, it is difficult to adequately cover this wide range of train speeds with a combination of actual superelevation and cant deficiency.

4.1.1 Maximum Train Speed versus Actual Train Speed

As suggested by Figure 10, even within a given train type, a range of actual train speeds below the maximum allowable speed for that train type may be observed. When this range of train speeds below maximum becomes wide, a design optimized for the maximum train speed may not adequately support the slowest trains. Depending on the exact frequency distribution of train speeds, a lower design speed may be more appropriate even if it means certain trains operate with additional cant deficiency.

Reasons for the existence of a range of actual train speeds below the maximum include:

- Grades that substantially reduce train speeds in the ascending direction.
- Certain trains have insufficient tractive effort or horsepower to sustain maximum speed.
- Individual railcars or commodities in the train consist may be subject to speed restrictions.
- Train speed may be limited by signal indications.
- Weather conditions may dictate slower speeds.
- Operating crews may have different train-handling styles.
- Operators reduce overall train speed to save fuel.

4.1.2 Site-Specific Factors

In addition to the above factors, there are certain locations where it is likely that a range of different operating speeds will be observed. Particular care should be taken when designing curve superelevation within typical train acceleration and braking distances of the following locations where trains frequently operate well below the posted maximum train speed:

- Terminals, passing sidings, or interlockings where trains slow down to negotiate turnouts
- Stations where local passenger trains stop and express trains do not stop
- Track sections adjacent to civil speed restrictions (including other curves)
- Spur connections to rail customers or local switching areas
- Crew change points
- Segments subject to congestion and train delay

4.2 Running Time and Curve Speed Restrictions

As the cant deficiency of a train on a particular curve increases, running time is reduced. For example, on a curve of fixed length and actual superelevation of 3 inches, the time taken for a typical regional intercity passenger train to traverse the curve at different amounts of cant deficiency can be calculated (Figure 11). The calculation includes allowances for deceleration and acceleration between a track speed of 79 mph and the maximum allowable curve speed corresponding to each level of cant deficiency. Time taken to traverse the curve is calculated for different cant deficiencies and different degree of curvature. Differences in running time (and running time penalties of speed restrictions) are magnified on higher-degree curves where the maximum curving speed is more sensitive to allowable cant deficiency. For peak overall corridor performance, it is often better to eliminate the largest speed restrictions on the slowest curves than to make small incremental speed improvements on faster curves where trains already operate close to the maximum speed on tangent track (Caughron et al. 2013). On faster curves, a small speed restriction for faster trains may be justified if it allows slower freight trains to operate with cant deficiency instead of with cant excess and its associated maintenance concerns.

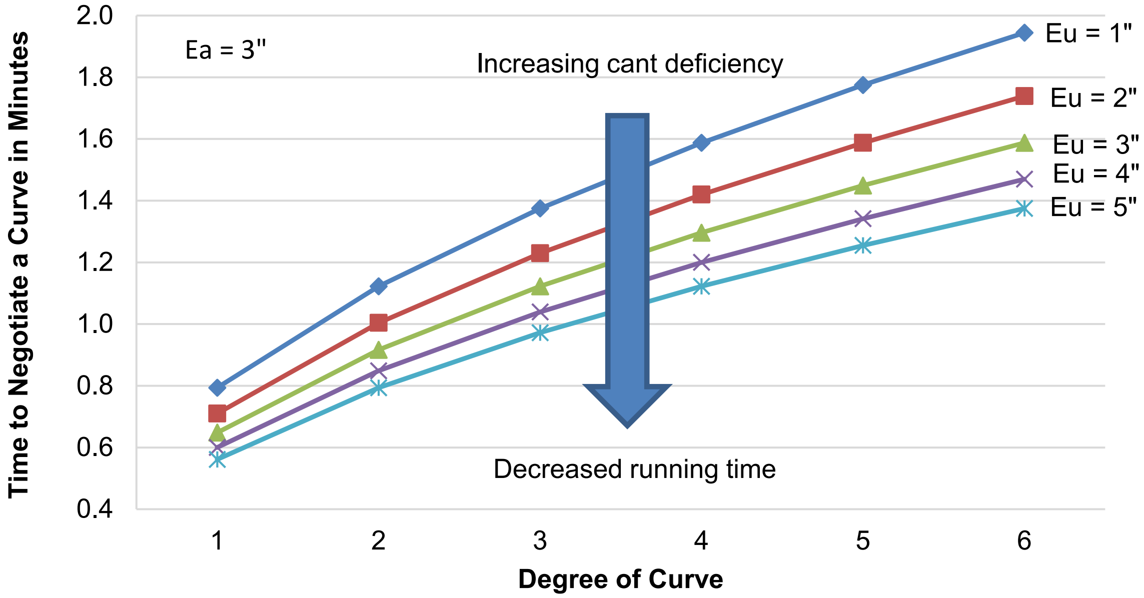


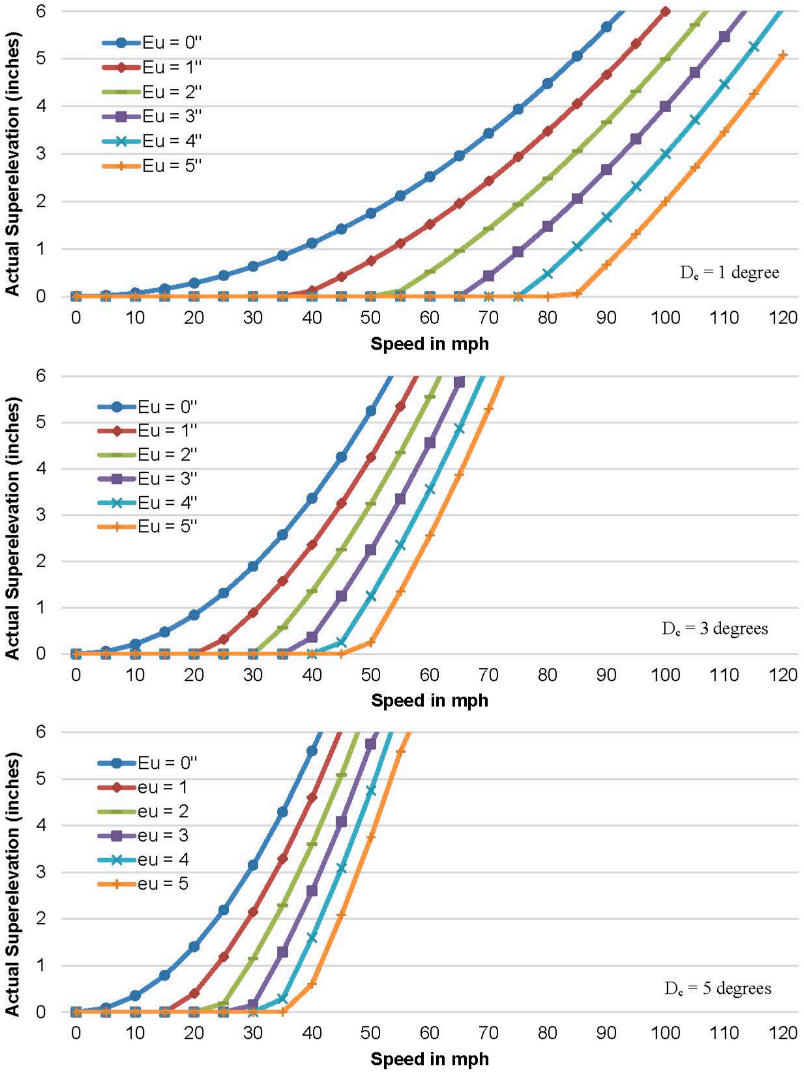
Figure 11: Example running-time benefits of increased cant deficiency on curves

4.3 Superelevation Bandwidth

For a given degree of curvature and actual superelevation (E_a), Equation 4 indicates the maximum train speed is dictated by the allowable cant deficiency (E_u). Increasing cant deficiency facilitates faster train speeds. As discussed in the earlier maintenance section, a consensus of most papers is that the overbalance condition is undesirable due to wear on the low rail. Thus the balanced speed (as calculated with Equation 4 where $E_u = 0$) sets a desirable minimum train speed for the curve. The conditions of $E_u = 0$ and $E_u =$ Maximum E_u set the lower and upper bounds on a "bandwidth" of desirable speeds to negotiate a curve with given actual superelevation (Igwemezie, 2006; Tournay et al., 2014a-c).

Since the required superelevation and cant deficiency increases with the square of the speed, the width of the superelevation bandwidth decreases as desired maximum speed increases (Figure 12). Finding a combination of superelevation and cant deficiency with sufficient bandwidth to cover a range of operating speeds is more difficult as maximum train speed increases.

For a given degree of curvature, although maximum speed increases, superelevation bandwidth decreases as actual superelevation is increased. Finally, as the degree of curvature increases, superelevation bandwidth decreases rapidly. On sharper curves, operating with high cant deficiency is less effective at increasing maximum train speeds compared to shallower curves.



D_c = 1 degree

D_c = 3 degrees

D_c = 5 degrees

Figure 12: Range of speeds between minimum balanced speed for freight and passenger speed at maximum cant deficiency (a) 1-degree curve, (b) 3-degree curve, and (c) 5-degree curve

4.4 Bandwidth and Distribution of Train Speeds: Graphical Approach

Since superelevation bandwidth is defined over a range of train speeds, it can be directly overlaid on a train speed frequency distribution (Figure 13). Combining these two figures provides a visual framework for selecting the optimal combination of actual superelevation and cant deficiency to cover the majority of trains in the train speed distribution.

For the example of a 1-degree curve with 1 inch of actual superelevation and 5 inches of cant deficiency, the corresponding superelevation bandwidth A can be directly compared to the train speed frequency distribution (Figure 13a). Trains falling below the lower bound are freight trains that will operate on the curve in the undesirable overbalance condition. Trains falling above the upper bound will be subject to civil speed restrictions that increase running time (and effectively create built-in train delay). To avoid maintenance issues and delaying trains, the number of trains in each "tail" outside bandwidth A should be minimized.

Using this graphical approach, the effectiveness of different superelevation solutions can be compared. Continuing with the example, increasing actual superelevation to 2 inches while maintaining 5 inches of cant deficiency increases maximum passenger train speed, resulting in fewer delayed trains (Figure 13b). However, the decrease in overall superelevation bandwidth B and increased maximum speed result in the lower bound speed increasing from 40 to 55 mph. At this equilibrium speed, the majority of freight trains will operate in the undesirable overbalance condition.

A third superelevation solution with actual superelevation of 4 inches and cant deficiency of 3 inches (Figure 13c) maintains the same maximum speed as bandwidth B. However, with lower cant deficiency and greater actual superelevation, bandwidth C is much narrower and all trains operating below 75 mph will experience overbalance conditions.

In the following section, this graphical technique is developed into a more formal mathematical model to optimize superelevation.

4.5 Superelevation Optimization Framework

Since the possible combinations of actual superelevation and cant deficiency is limited, the graphical approach of matching superelevation bandwidth to a train speed frequency distribution is effective at suggesting a superelevation design for a single curve or a small number of curves. However, it could become cumbersome for a route with many curves or a complex distribution of train speeds. To facilitate rapid optimization over many curves, a formal mathematical model for optimizing superelevation has been formulated.

4.5.1 Mathematical Model

As depicted graphically in Figure 13, the bandwidth concept seeks to maximize the number of trains (or equivalent tonnage or number of passengers) that fall between the equilibrium speed and speed at maximum cant deficiency. This is also equivalent to minimizing the number of trains falling outside the bandwidth. By dividing the train speed distribution into a series of discrete train speed groups, optimizing superelevation bandwidth can be formulated as a "set-covering problem." Such problems are often solved with a mixed-integer program (MIP). However, since superelevation and train speed are related quadratically, the problem is formulated as a mixed-integer quadratically constrained program (MIQCP).

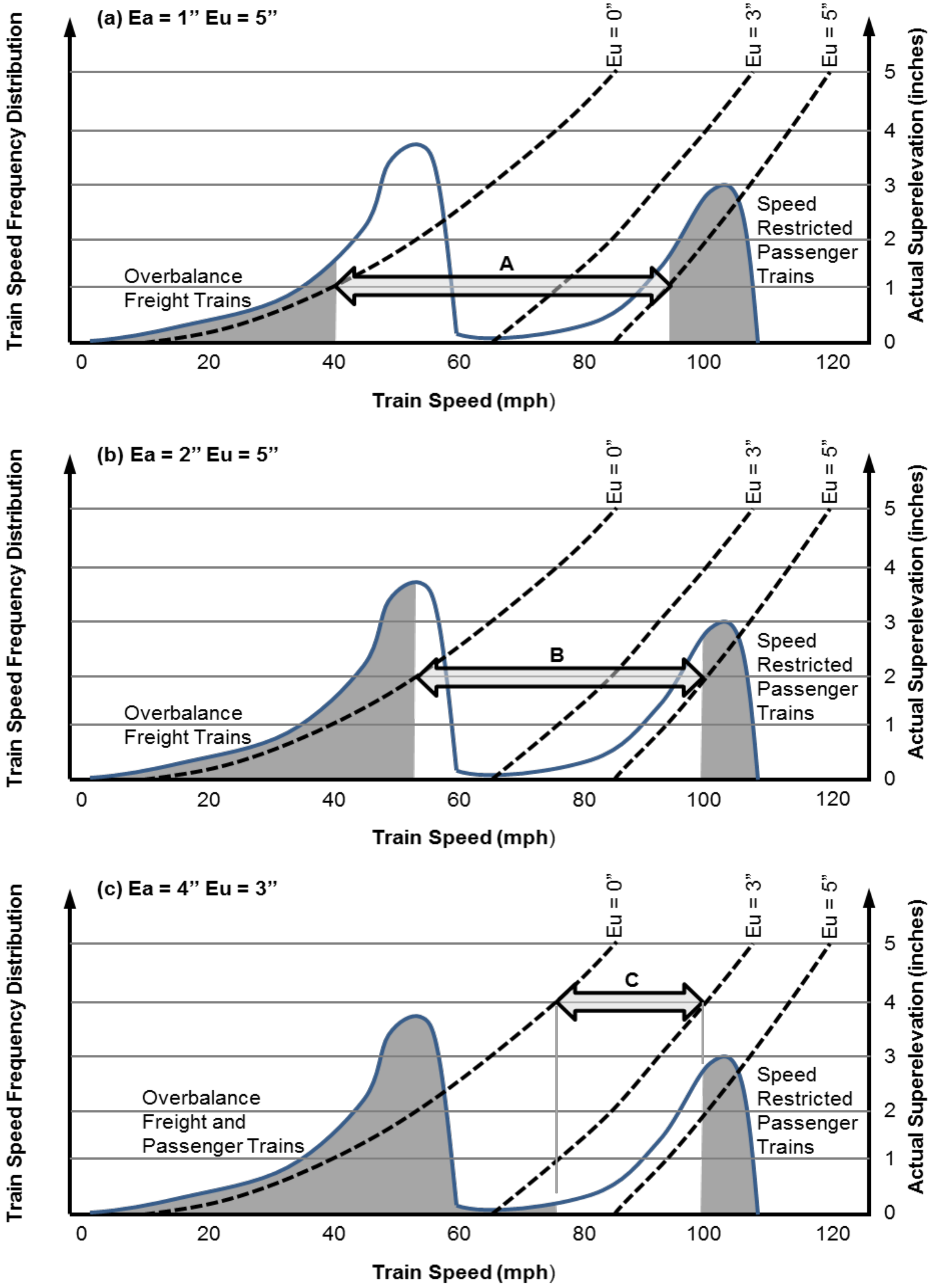


Figure 13: Portion of train speed frequency distribution covered by the speed bandwidth corresponding to different combinations of Ea and Eu on a 1-degree curve

Equations 9 through 17 describe the form of the model:

$$Minimize\ Z = \beta \sum_i S_{fi} G_i + \gamma \sum_i S_{pi} T_i \quad (9)$$

$$Subject\ to$$

$$E_a + E_u = 0.0007 D V_{max}^2 \quad (10)$$

$$E_a = 0.0007 D V_{min}^2 \quad (11)$$

$$V_{min} - V_i \leq S_{fi} V_{top} \qquad \forall i \quad (12)$$

$$V_i - V_{max} \leq S_{pi} V_{top} \qquad \forall i \quad (13)$$

$$E_{amin} \leq E_a \leq E_{amax} \quad (14)$$

$$0 \leq E_u \leq E_{umax} \quad (15)$$

$$S_{fi} = 0,1 \qquad \forall i \quad (16)$$

$$S_{pi} = 0,1 \qquad \forall i \quad (17)$$

Where:

D = degree of curvature

E_a = actual superelevation (design)

E_u = cant deficiency (design)

E_{amin} = minimum actual superelevation allowed by design criteria

E_{amax} = maximum actual superelevation allowed by design criteria

E_{umax} = maximum cant deficiency allowed by design criteria

V_{min} = minimum bandwidth speed

V_{max} = maximum bandwidth speed

V_{top} = fastest observed train speed

V_i = median speed of trains in speed group i

G_i = annual million gross tons of freight traffic in speed group i

T_i = annual passenger traffic in speed group i

β = coefficient quantifying cost of 1 MGT of freight train operation in overbalance condition

γ = coefficient quantifying revenue loss of increased passenger running time per passenger

S_{fi} = freight binary variable for speed group i (0 if freight traffic in bandwidth, 1 if outside bandwidth)

S_{pi} = passenger binary variable for speed group i (0 if passenger traffic in bandwidth, 1 if outside)

When solved, the model will provide the design values of E_a and E_u that minimize the “cost” of trains operating outside the superelevation bandwidth. Equation 9, the objective function, minimizes the weighted sum of MGT of freight traffic falling below the minimum bandwidth speed and of passenger traffic falling above the maximum bandwidth speed. Equation 10 defines the maximum bandwidth speed according to the actual superelevation and cant deficiency design. Equation 11 defines the minimum bandwidth speed as the equilibrium speed for the actual superelevation design. Equations 12 and 13 determine if speed group *i* falls within the superelevation bandwidth and set the freight and passenger binary S_{fi} and S_{pi} variables accordingly. Equations 14 and 15 ensure the design actual superelevation and cant deficiency are positive and do not exceed the allowable values specified in the design criteria. Equations 16 and 17 define the freight and passenger binary variables for all speed groups.

4.5.2 Implementation

One compromise of this model formulation is that actual superelevation can assume any real value between the minimum and maximum superelevation allowable under the railway design criteria. In practice, actual superelevation is installed in even increments of 0.25 or 0.125 inches. The solution provided by the model must be rounded to the nearest feasible value of actual superelevation. Reformulating the model to only consider even increments of actual superelevation is possible but would greatly increase the number of integer variables and make it more difficult to solve.

Of practical concern in implementing this mathematical model as a superelevation optimization framework is selection of the coefficients β and γ. Although many research papers document the negative maintenance implications of operating in an overbalance condition, none quantify the incremental maintenance expense as a function of freight traffic. Zarembski and Patel (2010) presented a detailed model of the cost of additional track maintenance for passenger trains on freight corridors but it does not include unbalance or superelevation as specific parameters. Additional research is needed to select values of β that directly relate to maintenance costs. Passenger operators may have a good feel for the revenue implications of imposing civil curve speed restrictions on passenger trains but in the absence of data, setting γ also requires engineering judgement. To simplify implementation of the model, the coefficients can also be set to simply reflect the relative importance of passenger and freight trains to the railroad operator and weight their influence on the superelevation design accordingly.

5. Case Studies

To demonstrate how various combinations of train traffic can result in different superelevation designs on mixed-use corridors, this section examines similar degree curves on five different case study corridors. The conditions of each case study are representative of actual mixed-use corridors, ranging from lines with predominantly freight traffic and infrequent passenger service to passenger corridors with different classes of passenger service and limited freight operations:

- Freight corridor with passenger traffic
- Freight corridor with advanced passenger trainset
- Freight corridor with higher-speed passenger trains
- Higher-speed passenger corridor with local freight service
- High-speed passenger corridor with local freight service

Each case study follows specific design criteria appropriate for the type of mixed-use corridor operation. However, to facilitate comparisons between case study corridors, common degree curves are used for all case studies instead of site-specific curve locations.

The superelevation designs for each case study are summarized in a table. Each table is divided into sections corresponding to a different degree of curvature. An example of one section of a table is shown below (Table 5). The first column displays the different train types with their corresponding speeds in the second column and maximum allowable cant deficiency in the third column. A train type may have more than one row if it is evaluated at different curving speeds (i.e., subject to a curve speed restriction). The fourth column displays the equilibrium superelevation for the train speed on that degree of curve. The five columns at right display the amount of cant deficiency the train is subject to when operating on the specified degree of curve, at the speed in Column 2, and with the actual superelevation in the column header. For example, if train type A operates on the 3-degree curve at 55 mph with 3.5 inches of actual superelevation, it is subject to 2.85 inches of cant deficiency. Negative values indicate cant excess. If the cant deficiency exceeds the allowable limit for that train type, the value is *italicized* to indicate an infeasible design combination. **Bold** values indicate the combination of actual superelevation, train speeds, and cant deficiency recommended for implementation. In the example below, 3.5 inches of superelevation are installed, and train type A is subject to a 55-mph speed restriction on this curve.

Table 5. Example corridor: cant deficiency of different train types

Degree of Curve:	3	**Superelevation - E_a:**		3.00	3.25	**3.50**	3.75	4.00
Train Type	**Speed (mph)**	**Max. E_u (inches)**	**E_e (inches)**	**Cant Deficiecy - E_u (inches)**				
Train Type A	60	3.00	7.56	*4.56*	*4.31*	*4.06*	*3.81*	*3.56*
Train Type A	**55**	3.00	6.35	*3.35*	*3.10*	**2.85**	2.60	2.35
Train Type B	**50**	2.00	5.25	*2.25*	2.00	**1.75**	1.50	1.25
Train Type C	**40**	2.00	3.36	0.36	0.11	**-0.14**	-0.39	-0.64

5.1 Freight Corridor with Passenger Traffic

The first case study is representative of a typical freight corridor with limited passenger service. The majority of traffic is freight operations. Most freight trains operate at or near track speed of 50 mph but a small number of local freight trains serving online customers often operate at speeds below 30 mph. Two long-distance passenger trains operate on the corridor each day with a maximum tangent track speed of 79 mph.

Details of the train traffic for this case study are as follows:

- Long-distance (LD) passenger trains:
    - 79 mph tangent speed
    - Maximum 3 inches of cant deficiency
- Priority freight trains:
    - 50 mph tangent speed
    - Maximum 2 inches cant deficiency
- Local freight trains
    - 30 mph tangent speed
    - Maximum 2 inches cant deficiency

The freight railroad design criteria limit actual superelevation to 4 inches. Different superelevation designs for 1-, 3-, and 5-degree curves were considered (Table 6).

On the 1-degree curve, all trains can operate at their target speed. With maximum 3 inches of cant deficiency, passenger trains require a minimum of 1.5 inches of actual superelevation. With this amount of superelevation, the priority freight trains operate with 0.25 inch of cant deficiency. However, the local freight trains operate in the undesirable overbalance condition.

On the 3-degree curve with 3.5 inches of actual superelevation, passenger trains are subject to a 55-mph speed restriction. Passenger train speeds above 55 mph are not possible without violating the limits of 4 inches of actual superelevation or 3 inches of cant deficiency. With 3.5 inches of superelevation, priority freight trains operate with 1.75 inches of cant deficiency. However, local freight trains operate in the undesirable overbalance condition.

On the 5-degree curve, both the passenger trains and priority freight trains are subject to a 40-mph speed restriction. The passenger trains cannot achieve 45 mph without exceeding the limits on actual superelevation and cant deficiency. At 40 mph, priority freight trains require 3.75 inches of superelevation, governing the superelevation design for the curve. Because of their extra allowable cant deficiency, passenger trains could actually operate at 40 mph with less actual superelevation. At 3.75 inches of actual superelevation, local freight trains operate with 0.6 inch of cant excess. Eliminating this mild overbalance condition would require imposing further speed restrictions on priority freight trains.

Table 6. Freight corridor with passenger traffic: cant deficiency

Degree of Curve:	1	**Superelevation - E_a:**		1.00	1.25	**1.50**	1.75	2.00
Train Type	**Speed (mph)**	**Max. E_u (inches)**	**E_e (inches)**	**Cant Deficiecy - E_u (inches)**				
LD Passenger	**79**	3.00	4.37	*3.37*	*3.12*	**2.87**	2.62	2.37
Priority Freight	**50**	2.00	1.75	0.75	0.50	**0.25**	0.00	-0.25
Local Freight	**30**	2.00	0.63	-0.37	-0.62	**-0.87**	-1.12	-1.37
Degree of Curve:	3	**Superelevation - Ea:**		3.00	3.25	**3.50**	3.75	4.00
Train Type	**Speed (mph)**	**Max. E_u (inches)**	**E_e (inches)**	**Cant Deficiecy - E_u (inches)**				
LD Passenger	60	3.00	7.56	*4.56*	*4.31*	*4.06*	*3.81*	*3.56*
LD Passenger	**55**	3.00	6.35	*3.35*	*3.10*	**2.85**	2.60	2.35
Priority Freight	**50**	2.00	5.25	*2.25*	2.00	**1.75**	1.50	1.25
Local Freight	**30**	2.00	1.89	-1.11	-1.36	**-1.61**	-1.86	-2.11
Degree of Curve:	5	**Superelevation - Ea:**		3.00	3.25	3.50	**3.75**	4.00
Train Type	**Speed (mph)**	**Max. E_u (inches)**	**E_e (inches)**	**Cant Deficiecy - E_u (inches)**				
LD Passenger	45	3.00	7.09	*4.09*	*3.84*	*3.59*	*3.34*	*3.09*
LD Passenger	**40**	3.00	5.60	2.60	2.35	2.10	**1.85**	1.60
Priority Freight	**40**	2.00	5.60	*2.60*	*2.35*	*2.10*	**1.85**	1.60
Local Freight	**30**	2.00	3.15	0.15	-0.10	-0.35	**-0.60**	-0.85

Negative values of cant deficiency represent cant excess or overbalance condition.
Italicized values exceed maximum cant deficiency. **Bold values are the selected curve design parameters.**

5.2 Freight Corridor with Advanced Passenger Trainset

The second case study is representative of a freight corridor with two types of passenger service: a commuter operation with conventional passenger equipment and regional passenger service that uses advanced tilting trainsets capable of operation at higher cant deficiency. These conditions are similar to those in the Pacific Northwest where Amtrak uses Talgo trainsets to achieve faster speeds and reduced running time on a freight corridor with numerous curves.

Commuter and priority freight trains comprise the majority of the rail traffic. Most freight trains operate at or near track speed but a small number of local freight trains operate at speeds below 40 mph. Due to the frequency of their station stops, the commuter trains operate at lower speeds than the regional intercity tilting trains. Although they only represent a few round-trips on the corridor each day, the regional intercity tilting trains are very sensitive to running time.

Details of the train traffic for this case study are as follows:

- Regional passenger trains with advanced tilting equipment:
  - 79 mph tangent speed (limited by signal system)
  - Maximum 5 inches of cant deficiency

- Commuter trains with conventional equipment:
  - 60 mph tangent speed
  - Maximum 3 inches of cant deficiency
- Priority freight trains:
  - 50 mph tangent speed
  - Maximum 2 inches cant deficiency
- Local freight trains
  - 40 mph tangent speed
  - Maximum 2 inches cant deficiency

The freight railroad design criteria limit actual superelevation to 4 inches. Different superelevation designs for 1-, 3-, and 5-degree curves were considered (Table 7).

On the 1-degree curve with 0.5 inch of superelevation, all trains can operate at their target speed with cant deficiency (avoiding the overbalance condition). This is largely because the high cant deficiency of the advanced tilting trainset allows it to operate at 79 mph with less superelevation than conventional equipment in the first case study.

On the 3-degree curve with 3.5 inches of actual superelevation, the tilting trains are subject to a 60-mph speed restriction, and the commuter trains are restricted to 55 mph. Although tilting train speeds of 65 mph are theoretically possible, they would require maximum actual superelevation and maximum cant deficiency, a combination that violates Amtrak design criteria. The 55-mph commuter train speed at 3 inches of cant deficiency dictates the actual superelevation design of 3.5 inches. At 3.5 inches of actual superelevation, the local freight trains operate in a mild overbalance condition with 0.14 inch of cant excess.

On the 5-degree curve, both the commuter and priority freight trains are subject to a 40-mph speed restriction, while the tilting train is subject to a 50-mph speed restriction. Higher commuter and tilting train speeds exceed the limits on actual superelevation and cant deficiency. The design superelevation of 3.75 inches is dictated by the cant deficiency of both the tilting trains at 50 mph and freight trains at 40 mph. At these speeds, all trains operate with cant deficiency and avoid the overbalance condition.

Table 7. Freight corridor with advanced passenger trainset: cant deficiency

Degree of Curve:	1	**Superelevation - E_a:**		0.25	**0.50**	0.75	1.00	1.25
Train Type	**Speed (mph)**	**Max. E_u (inches)**	**E_e (inches)**	**Cant Deficiecy - E_u (inches)**				
Tilting Passenger	**79**	5.00	4.37	4.12	**3.87**	3.62	3.37	3.12
Commuter	**60**	3.00	2.52	2.27	**2.02**	1.77	1.52	1.27
Priority Freight	**50**	2.00	1.75	1.50	**1.25**	1.00	0.75	0.50
Local Freight	**40**	2.00	1.12	0.87	**0.62**	0.37	0.12	-0.13
Degree of Curve:	3	**Superelevation - E_a:**		3.00	3.25	3.50	3.75	4.00
Train Type	**Speed (mph)**	**Max. E_u (inches)**	**E_e (inches)**	**Cant Deficiecy - E_u (inches)**				
Tilting Passenger	65	5.00	8.87	*5.87*	*5.62*	*5.37*	*5.12*	4.87
Tilting Passenger	**60**	5.00	7.56	4.56	4.31	4.06	3.81	3.56
Commuter	60	3.00	7.56	*4.56*	*4.31*	*4.06*	*3.81*	*3.56*
Commuter	**55**	3.00	6.35	*3.35*	*3.10*	2.85	2.60	2.35
Priority Freight	**50**	2.00	5.25	*2.25*	2.00	1.75	1.50	1.25
Local Freight	**40**	2.00	3.36	0.36	0.11	-0.14	-0.39	-0.64
Degree of Curve:	5	**Superelevation - E_a:**		3.00	3.25	3.50	**3.75**	4.00
Train Type	**Speed (mph)**	**Max. E_u (inches)**	**E_e (inches)**	**Cant Deficiecy - E_u (inches)**				
Tilting Passenger	55	5.00	10.59	*7.59*	*7.34*	*7.09*	*6.84*	*6.59*
Tilting Passenger	**50**	5.00	8.75	*5.75*	*5.50*	*5.25*	**5.00**	4.75
Commuter	45	3.00	7.09	*4.09*	*3.84*	*3.59*	*3.34*	*3.09*
Commuter	**40**	3.00	5.60	2.60	2.35	2.10	**1.85**	1.60
Priority Freight	**40**	2.00	5.60	*2.60*	*2.35*	*2.10*	**1.85**	1.60
Local Freight	**40**	2.00	5.60	*2.60*	*2.35*	*2.10*	**1.85**	1.60

Negative values of cant deficiency represent cant excess or overbalance condition.
Italicized values exceed maximum cant deficiency. **Bold values are the selected curve design parameters.**

5.3 Freight Corridor with Higher-Speed Passenger Trains

This case study is representative of a freight corridor that has been upgraded for higher-speed (110 mph) passenger train operation and also hosts conventional long-distance passenger trains. These conditions are similar to those on the higher-speed rail corridor between Chicago, IL and St. Louis, MO and on route segments in New York where passenger trains operate at 110 mph.

Priority and local freight trains comprise the majority of the rail traffic. Most freight trains operate at or near track speed, but a small number of local freight trains operate at speeds below 30 mph. Two long-distance passenger trains operate on the corridor each day with equipment restricted to 90 mph. Although they only represent a few round-trips on the corridor each day, the higher-speed regional intercity passenger trains are very sensitive to running time.

Details of the train traffic for this case study are as follows:

- Regional higher-speed passenger trains:
    - 110 mph tangent speed
    - Maximum 5 inches of cant deficiency
- Long-distance passenger trains:
    - 90 mph tangent speed
    - Maximum 4 inches of cant deficiency
- Priority freight trains:
    - 50 mph tangent speed
    - Maximum 1 inch cant deficiency
- Local freight trains
    - 30 mph tangent speed
    - Maximum 1 inch cant deficiency

The freight railroad design criteria limit actual superelevation to 4 inches. Different superelevation designs for 1, 3 and 5-degree curves were considered (Table 8).

On the 1-degree curve, 110 mph requires 3.5 inches of superelevation and forces all freight trains to operate in the undesirable overbalance condition. Placing a 100-mph speed restriction on the regional higher-speed passenger trains reduces the required actual superelevation to 2 inches. With 2 inches of actual superelevation, the priority freight trains experience a mild overbalance condition while the local freight trains operate with 1.37 inches of cant excess.

On the 3-degree curve with 4 inches of actual superelevation, the regional trains are subject to a 65-mph speed restriction, the long-distance passenger trains are restricted to 60 mph, and the priority freight trains are restricted to 45 mph. The 65-mph regional intercity passenger train speed at 5 inches of cant deficiency dictates the actual superelevation design of 4 inches. At 4 inches of actual superelevation, the priority freight trains operate with cant deficiency but the local freights operate in the overbalance condition with over 2 inches of cant excess.

On the 5-degree curve with 3.75 inches of actual superelevation, the regional trains are subject to a 50-mph speed restriction, the long-distance passenger trains are restricted to 45 mph, and the priority freight trains are restricted to 35 mph. Higher train speeds exceed the limits on actual superelevation and cant deficiency. The design superelevation of 3.75 inches is dictated by the cant deficiency of both the higher-speed passenger trains at 50 mph and freight trains at 35 mph. Local freight trains operate in the overbalance condition with 0.60 inch of cant excess.

Table 8. Freight corridor with higher-speed passenger trains: cant deficiency

Degree of Curve:	1	**Superelevation - E_a:**		1.75	**2.00**	2.25	2.50	2.75
Train Type	**Speed (mph)**	**Max. E_u (inches)**	**E_e (inches)**	**Cant Deficiecy - E_u (inches)**				
HrSR Passenger	110	5.00	8.47	*6.72*	*6.47*	*6.22*	*5.97*	*5.72*
HrSR Passenger	**100**	5.00	7.00	*5.25*	**5.00**	4.75	4.50	4.25
LD Passenger	**90**	4.00	5.67	3.92	**3.67**	3.42	3.17	2.92
Priority Freight	**50**	1.00	1.75	0.00	**-0.25**	-0.50	-0.75	-1.00
Local Freight	**30**	1.00	0.63	-1.12	**-1.37**	-1.62	-1.87	-2.12
Degree of Curve:	3	**Superelevation - E_a:**		3.00	3.25	3.50	3.75	**4.00**
Train Type	**Speed (mph)**	**Max. E_u (inches)**	**E_e (inches)**	**Cant Deficiecy - E_u (inches)**				
HrSR Passenger	70	5.00	10.29	*7.29*	*7.04*	*6.79*	*6.54*	*6.29*
HrSR Passenger	**65**	5.00	8.87	*5.87*	*5.62*	*5.37*	*5.12*	**4.87**
LD Passenger	65	4.00	8.87	*5.87*	*5.62*	*5.37*	*5.12*	*4.87*
LD Passenger	**60**	4.00	7.56	*4.56*	*4.31*	*4.06*	3.81	**3.56**
Priority Freight	50	1.00	5.25	*2.25*	*2.00*	*1.75*	*1.50*	*1.25*
Priority Freight	**45**	1.00	4.25	*1.25*	1.00	0.75	0.50	**0.25**
Local Freight	**30**	1.00	1.89	-1.11	-1.36	-1.61	-1.86	**-2.11**
Degree of Curve:	5	**Superelevation - E_a:**		3.00	3.25	3.50	**3.75**	4.00
Train Type	**Speed (mph)**	**Max. E_u (inches)**	**E_e (inches)**	**Cant Deficiecy - E_u (inches)**				
HrSR Passenger	55	5.00	10.59	*7.59*	*7.34*	*7.09*	*6.84*	*6.59*
HrSR Passenger	**50**	5.00	8.75	*5.75*	*5.50*	*5.25*	**5.00**	4.75
LD Passenger	50	4.00	8.75	*5.75*	*5.50*	*5.25*	*5.00*	*4.75*
LD Passenger	**45**	4.00	7.09	*4.09*	3.84	3.59	**3.34**	3.09
Priority Freight	40	1.00	5.60	*2.60*	*2.35*	*2.10*	*1.85*	*1.60*
Priority Freight	**35**	1.00	4.29	*1.29*	*1.04*	0.79	**0.54**	0.29
Local Freight	**30**	1.00	3.15	0.15	-0.10	-0.35	**-0.60**	-0.85

Negative values of cant deficiency represent cant excess or overbalance condition.
Italicized values exceed maximum cant deficiency. **Bold values are the selected curve design parameters.**

5.4 Higher-Speed Passenger Corridor with Local Freight Service

This case study is representative of a state-owned passenger corridor that has been upgraded for higher-speed (110 mph) passenger train operation and also hosts local freight service. These conditions are similar to the segments of the higher-speed rail corridor in Michigan operating at 110 mph.

Regional intercity higher-speed passenger trains comprise the majority of the rail traffic. These trains are very sensitive to running time. Only two local freight trains operate on the corridor each day, so maintenance of the overbalance condition is less of a concern than on the case study corridors with more freight traffic.

Details of the train traffic for this case study are as follows:

- Regional higher-speed passenger trains:
  - 110 mph tangent speed
  - Maximum 5 inches of cant deficiency
- Local freight trains
  - 30 mph tangent speed
  - Maximum 2 inches cant deficiency

The railroad design criteria limit actual superelevation to 4 inches. Different superelevation designs for 1-, 3-, and 5-degree curves were considered (Table 9).

On the 1-degree curve, regional higher-speed passenger train operation at 110 mph requires 3.5 inches of actual superelevation. At 30 mph, the two local freight trains operate with 2.87 inches of cant excess. Since there are few freight trains, this overbalance condition is tolerated by the owning agency to avoid any passenger train speed restriction and minimize the running time of passenger trains over the corridor.

On the 3-degree curve with 4 inches of actual superelevation, the regional higher-speed passenger trains are subject to a 65-mph speed restriction. Operation at higher speeds exceeds the limits on actual superelevation and cant deficiency. At 4 inches of actual superelevation, the two local freight trains operate in the overbalance condition with over two inches of cant excess.

On the 5-degree curve with 3.75 inches of actual superelevation, the regional higher-speed passenger trains are subject to a 50-mph speed restriction. Higher train speeds exceed the limits on actual superelevation and cant deficiency. At the design superelevation of 3.75 inches, the local freight trains operate in the overbalance condition with 0.60 inch of cant excess.

Table 9. Higher-speed passenger corridor with local freight trains: cant deficiency

Degree of Curve:	1	**Superelevation - E_a:**		3.00	3.25	**3.50**	3.75	4.00
Train Type	**Speed (mph)**	**Max. E_u (inches)**	**E_e (inches)**	**Cant Deficiecy - E_u (inches)**				
HrSR Passenger	110	5.00	8.47	*5.47*	*5.22*	**4.97**	4.72	4.47
Local Freight	**30**	2.00	0.63	-2.37	-2.62	**-2.87**	-3.12	-3.37
Degree of Curve:	3	**Superelevation - E_a:**		3.00	3.25	3.50	3.75	**4.00**
Train Type	**Speed (mph)**	**Max. E_u (inches)**	**E_e (inches)**	**Cant Deficiecy - E_u (inches)**				
HrSR Passenger	70	5.00	10.29	*7.29*	*7.04*	*6.79*	*6.54*	*6.29*
HrSR Passenger	**65**	5.00	8.87	*5.87*	*5.62*	*5.37*	*5.12*	**4.87**
Local Freight	**30**	2.00	1.89	-1.11	-1.36	-1.61	-1.86	**-2.11**
Degree of Curve:	5	**Superelevation - E_a:**		3.00	3.25	3.50	**3.75**	4.00
Train Type	**Speed (mph)**	**Max. E_u (inches)**	**E_e (inches)**	**Cant Deficiecy - E_u (inches)**				
HrSR Passenger	55	5.00	10.59	*7.59*	*7.34*	*7.09*	*6.84*	*6.59*
HrSR Passenger	**50**	5.00	8.75	*5.75*	*5.50*	*5.25*	**5.00**	4.75
Local Freight	**30**	2.00	3.15	0.15	-0.10	-0.35	**-0.60**	-0.85

Negative values of cant deficiency represent cant excess or overbalance condition.
Italicized values exceed maximum cant deficiency. **Bold values are the selected curve design parameters.**

5.5 High-Speed Passenger Corridor with Local Freight Service

The final case study is representative of a passenger corridor where multiple types of passenger service, including high-speed trains, operate along with a small number of local freight trains. These conditions are similar to various segments of the Northeast Corridor between Boston, New York, and Washington, DC, where the Acela Express operates at speeds of 150 mph.

In this case study, regional passenger trains and commuter trains comprise the majority of the rail traffic. Due to the frequency of their station stops, the commuter trains have a lower maximum tangent track speed than regional passenger trains. Regional passenger trains also use rolling stock qualified to operate with an extra inch of cant deficiency compared to the commuter trains. High-speed express passenger trains also operate on the corridor with advanced tilting trainsets qualified to operate at higher levels of cant deficiency than conventional equipment. Although they only represent a few round-trips on the corridor each day, the high-speed express passenger trains are very sensitive to running time. Only two local freight trains operate on the corridor each day.

Details of the train traffic for this case study are as follows:

- High-speed express passenger trains with advanced trainsets:
    - 150 mph tangent speed
    - Maximum 7 inches of cant deficiency

- Regional passenger trains:
  - 125 mph tangent speed
  - Maximum 5 inches of cant deficiency
- Commuter trains:
  - 70 mph tangent speed
  - Maximum 4 inches cant deficiency
- Local freight trains
  - 40 mph tangent speed
  - Maximum 1.5 inches cant deficiency

Amtrak design criteria limit actual superelevation to 4 inches where freight trains are operated. Different superelevation designs for 0.5-, 1-, and 1.5-degree curves were considered (Table 10).

On the 0.5-degree curve, due to its allowable cant deficiency of 7 inches, 150 mph operation of the high-speed express trainset requires a minimum 1 inch of actual superelevation. At 1 inch of superelevation, the regional passenger and commuter trains can operate at their target speeds with allowable cant deficiency. The local freight trains operate with 0.44 inch of cant excess.

On the 1-degree curve with 4 inches of actual superelevation, the high-speed express trains are subject to a 125-mph speed restriction, and the regional passenger trains are restricted to 110 mph. The 125-mph high-speed express passenger train speed at 7 inches of cant deficiency dictates the actual superelevation design of 4 inches. At 4 inches of actual superelevation, the commuter trains and local freight trains operate in the overbalance condition with cant excess. To avoid the overbalance condition for the commuter trains, the curving speed of both the express and regional passenger trains would need to be decreased an additional 5 mph (to 120 mph and 105 mph, respectively) and superelevation decreased to 3.25 inches. This design change would also decrease the amount of cant excess for the local freight trains.

Table 10. High-speed passenger corridor with local freight trains: cant deficiency

Degree of Curve:	0.5	**Superelevation - E_a:**		0.25	0.50	0.75	**1.00**	1.25
Train Type	**Speed (mph)**	**Max. E_u (inches)**	**E_e (inches)**	**Cant Deficiecy - E_u (inches)**				
Express Passenger	**150**	7.00	7.88	*7.63*	*7.38*	*7.13*	**6.88**	6.63
Regional Passenger	**125**	5.00	5.47	*5.22*	4.97	4.72	**4.47**	4.22
Commuter	**70**	4.00	1.72	1.47	1.22	0.97	**0.72**	0.47
Local Freight	**40**	1.50	0.56	0.31	0.06	-0.19	**-0.44**	-0.69

Degree of Curve:	1	**Superelevation - E_a:**		3.00	3.25	3.50	3.75	**4.00**
Train Type	**Speed (mph)**	**Max. E_u (inches)**	**E_e (inches)**	**Cant Deficiecy - E_u (inches)**				
Express Passenger	130	7.00	11.83	*8.83*	*8.58*	*8.33*	*8.08*	*7.83*
Express Passenger	**125**	7.00	10.94	*7.94*	*7.69*	*7.44*	*7.19*	**6.94**
Express Passenger	120	7.00	10.08	*7.08*	6.83	6.58	6.33	6.08
Regional Passenger	115	5.00	9.26	*6.26*	*6.01*	*5.76*	*5.51*	*5.26*
Regional Passenger	**110**	5.00	8.47	*5.47*	*5.22*	4.97	4.72	**4.47**
Regional Passenger	105	5.00	7.72	4.72	4.47	4.22	3.97	3.72
Commuter	**70**	4.00	3.43	0.43	0.18	-0.07	-0.32	**-0.57**
Local Freight	**40**	1.50	1.12	-1.88	-2.13	-2.38	-2.63	**-2.88**

Degree of Curve:	1.5	**Superelevation - E_a:**		3.00	3.25	3.50	**3.75**	4.00
Train Type	**Speed (mph)**	**Max. E_u (inches)**	**E_e (inches)**	**Cant Deficiecy - E_u (inches)**				
Express Passenger	105	7.00	11.58	*8.58*	*8.33*	*8.08*	*7.83*	*7.58*
Express Passenger	**100**	7.00	10.50	*7.50*	*7.25*	7.00	**6.75**	6.50
Regional Passenger	95	5.00	9.48	*6.48*	*6.23*	*5.98*	*5.73*	*5.48*
Regional Passenger	**90**	5.00	8.51	*5.51*	*5.26*	*5.01*	**4.76**	4.51
Commuter	**70**	4.00	5.15	2.15	1.90	1.65	**1.40**	1.15
Local Freight	**40**	1.50	1.68	-1.32	-1.57	-1.82	**-2.07**	-2.32

Negative values of cant deficiency represent cant excess or overbalance condition.
Italicized values exceed maximum cant deficiency. **Bold values are the selected curve design parameters.**

On the 1.5-degree curve with 3.75 inches of actual superelevation, the high-speed express trains are subject to a 100-mph speed restriction, and the regional passenger trains are restricted to 90 mph. For both of these train types, higher train speeds exceed the limits on actual superelevation and cant deficiency. Increasing actual superelevation to 4 inches does not allow for a 5-mph increase in curving speed for either. The design superelevation of 3.75 inches is dictated by the cant deficiency of both the high-speed express passenger trains at 100 mph and regional passenger trains at 90 mph. Commuter trains are not subject to a speed restriction and can operate at 70 mph with 1.40 inches of cant deficiency. Local freight trains operate in the overbalance condition with 2.07 inches of cant excess.

6. Summary

6.1 Summary of Findings

Many factors influence the design of curve superelevation on mixed-use railway lines where different types of trains operate at different speeds. Disparity in train speeds may be due to the business objectives of certain types of freight and passenger service, the curving capability of the various types of rail vehicles operating over the line, and local site conditions that may cause certain trains to negotiate nearby curves at less-than-normal timetable operating speeds. Grades and the resulting in-train forces experienced by long freight trains present additional challenges for setting superelevation to avoid excessive lateral wheel/rail forces. Since the actual superelevation in track is fixed, under conditions of varying train speeds and in-train forces, different trains will experience cant deficiency or cant excess. Under these conditions, quasi-static vertical wheel loads can be increased by over 20 percent, increasing maintenance. In particular, the overbalance condition should be avoided, as it promotes rolling contact fatigue.

On a given curve, a design superelevation bandwidth can be defined and compared to the frequency distribution of train speeds operating on the route. Trains falling below the lower bound set by the equilibrium speed will operate in the undesirable overbalance condition. Trains above the upper bound speed set by maximum cant deficiency will be subject to civil curve speed restrictions. A combination of actual superelevation and allowable cant deficiency that satisfies railway design criteria while maximizing the number of trains falling within the superelevation bandwidth will provide the best solution for a mixed-use corridor. Using the bandwidth approach, trains can be weighted by traffic in terms of freight tonnage or passenger number to better reflect the maintenance and revenue implications of operating outside the design superelevation bandwidth.

The five case studies examined the challenges of establishing superelevation on curves for a variety of operating scenarios by looking at a limited number of possible curvature-superelevation combinations. A more comprehensive approach for evaluating an entire route or portion of a route would require additional field data to determine the bandwidth and frequency distribution of train speeds described in Section 4.4, as well as the detailed curve geometry data for all of the curves to be input into the model described in Section 4.5. Once the optimal superelevation is determined for each curve, the designer will have to verify that the spirals are of sufficient length to accommodate the recommenced superelevation.

6.2 Research Needs

This document presents both a graphical framework and mathematical approach to identifying the optimal superelevation design parameters with the bandwidth approach. The optimization framework could be improved by additional research to quantify the specific maintenance cost of operating freight trains with cant excess. Although there are published reports that document increased wear and maintenance on curves and link it to improper superelevation, there do not appear to be studies that investigate this phenomenon in a detailed, quantitative manner. While inferences can be made from calculated changes in lateral and vertical wheel forces for different combinations of actual superelevation, cant deficiency, and degree of curvature, it does not appear that the resulting wear and maintenance effects have been studied in a controlled experiment. Also, the wheel forces calculated earlier in this document are for a single axle under quasi-static conditions. The calculations do not consider additional forces arising from the actual steering and angle of attack created by the curving action of two- and three-axle trucks under railcars and locomotives. The resulting forces may mitigate or exacerbate the effect of overbalance and underbalance conditions on wheel/rail forces. Recent advances in field instrumentation of track under revenue traffic have the potential to better characterize the actual forces imparted on the track structure under different superelevation conditions.

Because of this knowledge gap, it is difficult to quantify the rail wear and maintenance consequences of a specific amount of cant excess in the overbalance condition or cant deficiency in the underbalance condition. How many inches of each are acceptable and how many inches lead to problems are open research questions, along with what other factors influence the acceptable threshold. Additional knowledge of this parameter will allow for more effective practitioner decisions regarding the trade-off between increasing maximum passenger train speeds to eliminate civil speed restrictions while simultaneously increasing the percentage of freight trains operating in an overbalance condition. It will also provide information to help better understand the impact that each train type has on track deterioration and the associated maintenance costs.

7. References

American Railway Engineering and Maintenance-of-Way Association. (2003). *Practical Guide for Railway Engineering*, Chapter 6: Railway Track Design. Landover, MD: American Railway Engineering and Maintenance of Way Association.

American Railway Engineering and Maintenance-of-Way Association. (2015). *Manual for Railway Engineering*, Chapter 5, Section 3. Landover, MD: American Railway Engineering and Maintenance of Way Association, Landover.

Australian Rail Track Corporation Ltd. (2015). *Engineering (Track & Civil) Code of Practice*, Section 5: Track Geometry.

BNSF Railway. (2000). *BNSF Engineering Instructions*, Section 5: Track Geometry. Fort Worth, TX.

California High-Speed Rail Authority. (2008). *Alignment Standards for Shared Use Corridors*, TM 1.1.6.

Caughron, B.M., C. T. Dick, C. T., & Barkan, C.P.L. (2013). A project selection model for improving running time on passenger rail corridors. *Proceedings of the American Railway Engineering and Maintenance-of-Way Association Annual Conference*, Indianapolis, Indiana, September 2013.

CN Railway. (2002). *CN Engineering Recommended Method for Determining Superelevation, Spiral Length and Maximum Train Speed on Curves*, RM 1305-0.

CSX Transportation. (2002). *Standard Plan 2511: Superelevation of Curves*. Jacksonville, FL.

Elkins, J.A., & Gostling, R.J. (1977). A general quasi-static curving theory for railway vehicles. *Vehicle System Dynamics*, 6 (2-3), 100-106.

U.S. Code of Federal Regulations Title 49 Part 213 Subpart C Section 213.57.

Förstberg, J., Andersson, E., & Ledin, T. (1998). Influence of different conditions for tilt compensation on symptoms of motion sickness in tilting trains. *Brain Research Bulletin*, 47 (5), 525-535.

Harris, H.R., Schmid, E., & Smith, R.A. (1998). Introduction: Theory of tilting train behaviour. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 212 (1), 1-5.

Igwemezie, J.O. (2006). Re-engineering superelevation: Part 2 of 2. *Railway Track and Structures*, 102 (8).

Igwemezie, J., Marcks, D., Van Trump, W.E., & Clark, D.W. (2006). High rail wear on the Tehachapi: problem identification, analysis and implementation of solutions. *Proceedings of the American Railway Engineering and Maintenance-of-Way Association Annual Conference*, Louisville, Kentucky, September 2006.

Kollmar, R.A. (2006). *The Balancing Act: The Fine Art of Using Unbalance to Maximize the Efficiency of Railroad Infrastructure*. W.W. Hay Railroad Engineering Seminar Series presentation, University of Illinois at Urbana-Champaign, April 28, 2006.

Lindahl, M. (2001). *Track Geometry for High-Speed Railways: A Literature Survey and Simulation of Dynamic Vehicle Response*. Doctoral Thesis, KTH Royal Institute of Technology, Department of Vehicle Engineering. Stockholm, Sweden.

Massachusetts Bay Transportation Authority. (1996). *Commuter Rail Design Standards Manual*, Volume 1, Section 1: Track and Roadway. Boston, MA.

Moody, J.C. (2014). *Critical Speed Analysis of Railcars and Wheelsets on Curved and Straight Track*. Honors Thesis, Bates College, Lewiston, ME.

Norfolk Southern Railway. (2013). *Public Projects Manual*, Track Design Guide, Curve Superelevation. Atlanta, GA.

Oldknow, K.D., & Eadie, D.T. (2010). Top of rail friction control as a means to mitigate damaging lateral loads due to overbalanced operation of heavy axle load freight traffic in shared high speed rail corridors. *Proceedings of the 2010 Joint Rail Conference,* American Society of Mechanical Engineers.

Plotkin, D. (1997). Carrying freight on high-speed rail lines. *Journal of Transportation Engineering*, 123 (3), 199-201.

Roney, M. (2009). Reducing the stress state on Canadian Pacific's Western Corridor. *Interface: The Journal of Wheel/rail Interaction*.

Roney, M., Bell, S., Paradise, S., Oldknow, K., & Igwemezie, J. (2010). Implementation of distributed power and friction control to minimize the stress state and maximize velocity in Canadian Pacific's heavy haul/heavy grade operations. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 224 (5), 465-471.

Sadeghi, J., & Akbari, B. (2006). Field investigation on effects of railway track geometric parameters on rail wear. *Journal of Zhejiang University Science A*, 7 (11):,1846-1855.

Sadeghi, J., & Shoja, H. (2013). Impact of superelevation deficiencies on the loading pattern of railway sleepers. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 227 (3), 286-295.

Southern California Regional Rail Authority. (2014). *SCRRA Design Criteria Manual*. Los Angeles, CA.

Tournay, H., Cakdi, S., Trevithick, & Morrison, K. (2014a). The effect of track cant on vehicle curving: theory and single car test results, Part I of III. *Technology Digest*, TD-14-013. Transportation Technology Center, July 2014.

Tournay, H., M. Akhtar, S. Cakdi, & Morrison, K. (2014b). The effect of track cant on vehicle curving: in-service site selection and analysis, Part II of III. *Technology Digest*, TD-14-014. Transportation Technology Center, July 2014.

Tournay, H., M. Akhtar, S. Cakdi, & Morrison, K. (2014c). The effect of track cant on vehicle curving: in-service test results, Part III of III. *Technology Digest*, TD-14-015. Transportation Technology Center, July 2014.

Union Pacific Railroad. (2011). *Track Standard Drawings*, Standard Drawings 0019-0023. Omaha, NE.

Utah Transit Authority. (2010). *Utah Transit Authority Commuter Rail Design Criteria*, Chapter 3: Alignment. Salt Lake City, UT.

Vermeij, D., (2000). Design of a high speed track. *HERON Journal*, 45 (1).

Zarembski, A.M., & Patel, P. (2010). Estimating maintenance costs for mixed higher speed passenger and freight rail corridors. *Proceedings of the 2015 Joint Rail Conference*, American Society of Mechanical Engineers.

Abbreviations and Acronyms

Abbreviation or Acronym	Name
E_e	Actual Superelevation (Cant)
AREMA	American Railway Engineering and Maintenance-of-Way Association
BNSF	BNSF Railway
CP	Canadian Pacific Railway
CN	CN Railway
CSX	CSX Transportation
E_e	Equilibrium Superelevation
E_u	Unbalanced Superelevation (Cant Deficiency or Cant Excess)
MBTA	Massachusetts Bay Transportation Authority
NRPC	National Railroad Passenger Corporation (Amtrak)
NS	Norfolk Southern Railway
SEPTA	Southeastern Pennsylvania Transportation Authority
SCRRA	Southern California Regional Rail Authority (Metrolink)
E_u	Unbalanced Superelevation (Cant Deficiency or Cant Excess)
UP	Union Pacific Railroad
UTA	Utah Transit Authority